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Game theoretic modeling of prosumers

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SID: 3301130005

SCHOOL OF SCIENCE & TECHNOLOGY

A thesis submitted for the degree of

Master of Science (MSc) in Information and Communication Systems

NOVEMBER 2014

THESSALONIKI – GREECE

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Abstract

This dissertation was written as a part of the MSc in ICT Systems at the International Hellenic University. The topic of this paper is concentrated on the power grid and how it can be modeled when a small group of prosumers exchange electricity among themselves.

First we elaborate how the smart grid 3.0 works and its evolution from the smart grid. Here we concentrate on the architecture of the previous versions of the smart grid, as well as the role of ICT and renewable sources of energy (solar, wind). Next we explain the architecture of the entities of the contemporary smart grid, such as microgrids, virtual power plants, prosumers and transactive energy markets. In the last part of this chapter we explain the energy market, and its evolution from monopolized market to a deregulated market. The operation of electricity exchange on a wholesale and retail level is explained at this point.

In chapter 3 we elaborate the prosumer scenario we have developed by first starting with describing game theory in general. Next we explain the Shapley value and how we can use that model to allow users in our scenario to collaborate among each other and exchange electricity for benefits. We propose our solution and explain our findings. Next we propose a schematic model of distributing the remaining electricity to the grid by using again the Shapley theorem.

Acknowledgements:

Here I would like to express my gratitude to all the people that helped me to accomplish this master course and have an amazing year:

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Anja Ristovska

22/11/2014

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1 Introduction

What is the smart grid? There is no one definition about what it is, but there is a certain reason why there is a need for it. The emerging technologies and the opportunities they give for optimization and automation of the electricity network, gives a good incentive for its implementation. The so called third industrial revolution provides for each appliance, business and residence to be able to connect to the grid, in turn allowing users to communicate interactively, decide when to plug in their devices depending on what is the price per kWh at the given moment, something made possible by real time pricing schemes. This will be enabled by big data collection allowing users to make smart decisions about their electricity usage. It will also allow for more renewables to be used for electricity generation, using photovoltaic rooftop panels, small windmill parks and with the help of batteries and electronic vehicles, store this electricity and use it as needed, thus making households and other facilities, producers as well.

In general we can say that the *"Smart Grid is an intelligent system dedicated to ease the communication at different levels of the power grid, allow higher control over the system, increase energy delivery efficiency and enable electricity generation by prosumers at remote locations to be integrated within the existing power grid."*

The smart grid is a topic developed countries started becoming more and more interested in at the turn of the 21st century. With the expansion of technology, the ever increasing pollution and global warming issues, the possibility of using renewable 'clean' energy, the obsolescence of the centralized grid (which at the moment is working at its optimum, but with the ever increasing electricity demand, population increase and different demand during different periods, has become very costly), as well as in some cases, the fear of terrorist attack on the grid network, made the world leaders stop and think about one of the biggest necessities of the human race.

We can say that the smart meters are the first step towards a smarter grid, the so called Smart Grid 1.0. This is more or less the stage where we are now: smart meters are implemented in houses and businesses, collecting data about their electricity patterns of usage, without the need for human interaction with the device as it was before, when there was a need for an utility company employee to come and read the numbers manually.

Thesis structure

The objective of this thesis is to examine the possibility of electricity sharing among users in a small community. The users' collaboration is modeled with the Shapley theorem in order to fairly distribute benefits.

In Chapter 2 we elaborate the smart grid – its evolution, including smart grid 1.0 entities like smart meters and their role in the seamless communication between generators and consumers. The entities of smart grid 2.0 including real time pricing and demand response techniques are elaborated to some extent, providing examples of real example implementations in the grid. The advanced metering infrastructure together with the benefits of its implementation and the types of different loads in the household and which of them allow for DR techniques are detailed. Different ways of prosumer production from renewable energy sources (solar, wind) are introduced here. In the next part we investigate the smart grid entities, including microgrids with distributed generation, virtual power plants, prosumers with prosumer based architecture as well as the transactive energy market and the TeMIX protocol that allows for communication and transactive operarions of end devices (electricity users), transport entities and intermediaries. Next the energy market is explained in general, with the whole process of electricity generation, through distribution to usage. The basic architecture is explained, together with recommendations for a more reliable transmission to avoid blackouts. A basic notion is given on the energy market deregulation, together with examples of how energy is transacted with. Further insight is given on the wholesale market operation with more detailed elaboration of the types of markets and transmission of electricity on this level. Next we briefly explain how the retail market operates in deregulated markets, together with the enhanced services offered by utilities.

In Chapter 3 we start by explaining briefly the concept of game theory. Next we go into more depth in cooperative games, focusing on the Shapley theorem and the concept of the core of a game. Types of players are described briefly. The concepts of game superadditivity, subadditivity and monotonicity are explained in the next section, followed by the concept of the core. Here we give a simple example of a cooperative game, with computing the Shapley value and finding the core, if there is one. We see the difference between a stable and fair coalition. Next we present the notion of a convex game followed by an example of a convex game. Next we explain more in depth the Shapley theorem, derived from the Shapley axioms of symmetry, dummy players and additivity. We explain the concept of the grand coalition and present a simple 2 player game and how the total benefits are divided fairly among users. In the next part of this chapter we explain our proposed scenario of prosumers and how they will be modeled. Each user has a consumption level and some also produce electricity from different PV systems installed. We state the problem and which users will be modeled. Next we propose the algorithm and assume a pseudocode.

In Chapter 4 we elaborate our results. We do the calculations using the Shapley theorem, finding the Shapley value of each user in the game. We see whether the grand coalition is stable, and if not we form a new stable coalition set. We calculate the new values for each player and compare our findings. Next we assume each player changes consumption levels, while other players have a constant consumption. We have four different consumption changes for each player. We calculate new Shapley values and plot them on a graph to compare findings. Next we calculate amount of electricity that is more than needed in the society to be sold to the grid. We propose four pricing schemes and calculate benefits of players for each scheme. We compare results to find most beneficial one.

In Chapter 5 we summarize the conclusions of our findings.

2 The smart grid 3.0

2.1 Smart Grid - Introduction

What is the smart grid? There is no one definition about what it is, but there is a certain reason why there is a need for it. The emerging technologies and the opportunities they give for optimization and automation of the electricity network, gives a good incentive for its implementation. The so called third industrial revolution provides for each appliance, business and residence to be able to connect to the grid, in turn allowing users to communicate interactively, decide when to plug in their devices depending on what is the price per kWh at the given moment, something made possible by real time pricing schemes. This will be enabled by big data collection allowing users to make smart decisions about their electricity usage. It will also allow for more renewables to be used for electricity generation, using photovoltaic rooftop panels, small windmill parks and with the help of batteries and electronic vehicles, store this electricity and use it as needed, thus making households and other facilities, producers as well.

As it is now possible to share information over the internet, it will be made possible to share energy over the internet. As Ethernet inventor Robert Metcalfe said on his Enernet tour in 2008: “Over the past 63 years, we met the world needs for cheap and clean *information* by building the *Internet.* Over the next 63 years, we will meet world needs for cheap and clean *energy* by building the *Enernet.*” With this statement he announced a new era, an era of clean and affordable energy, one made possible by exploiting renewable electricity resources, located on remote locations closer to the user and interconnected and controlled via the already existing Internet.

The Smart Grid is defined similarly by different governing bodies.

So, according to the European SmartGrids Technology Platform,[1] *“A Smart Grid is an electricity network that can intelligently integrate the actions of all users connected to it – generators, consumers and those that do both [prosumers] – in order to efficiently deliver sustainable, economic and secure electricity supplies.”*

The US Department of Energy [2] states that *"A Smart Grid uses digital technology to improve reliability, security and efficiency (both economic and energy) of the electric system from large generation, through the delivery systems to electricity consumers and a growing number of distributed generation and storage resources."*

In UK's discussion paper from the Department of Energy called "Smarter Grids: The opportunity" [3] it is stated that *"the principles of a so called 'smarter' grid are allowing it to be observable (making it possible to have access to operational indicators at LV in real time), controllable (higher possibility for control and optimization of the power system), automated (a network with certain level of 'intelligence') and fully integrated (new components that are fully interoperable with the existing system as well as with new devices)"*

In general we can say that the *"Smart Grid is an intelligent system dedicated to ease the communication at different levels of the power grid, allow higher control over the system, increase energy delivery efficiency and enable electricity generation by prosumers at remote locations to be integrated within the existing power grid."*

One of the essential drivers of the development of the smart grid is the Distributed Generation (DG), meaning that energy is starting to be generated at the very end of the grid, rather than at the centralized generators. This in turn gives way from having only a couple of generation points that can be easily and centrally controlled, to thousands and eventually, millions of generation points, which will have to be coordinated in order to operate well together. And as energy is not a commodity that can be stored easily, the smart grid will allow for seamless exchange of energy among generators/consumers at the local level. Another reason is the communication among generators and users. So far the communication has been unidirectional, with data from users homes and facilities being sent to utilities central stations for demand forecasting and billing purposes. With the emergence of the smart grid it will be possible for consumers to communicate among themselves, exchange energy over transactive energy markets, receive information about electricity prices in real time and make smart decisions about thei consumption patterns based on this information.

2.1.1 Smart grid architecture

The general smart grid architecture consists of entities that must exist and function with interoperability within the grid in order to achieve results (they are represented below in Figure 1 Smart grid architecture):

- Renewable energy sources - such as solar (solar panels) and wind (wind parks);
- Smart buildings and households - smart in a way that a user can control and have access in real time of every aspect of the energy consumption;
- Smart meters – meters that aggregate data and give users information about electricity prices in real time;
- Electric vehicles – vehicles that run on electricity, are environment friendly and can be used for energy storage as well;
- Energy storage (batteries) – this is needed in order to store electricity that is produced close to/by the user for later usage in order to increase user comfort;
- Transmission and distribution network – in order to distribute electricity at larger distances and connect millions of production and consumption points;
- Smart appliances – ones that allow saving electricity

Figure 1 Smart grid architecture

2.2 Evolution from Smart Grid

The smart grid is a topic developed countries started becoming more and more interested in at the turn of the 21st century. With the expansion of technology, the ever increasing pollution and global warming issues, the possibility of using renewable ‘clean’ energy, the obsolescence of the centralized grid (which at the moment is working at its optimum, but with the ever increasing electricity demand, population increase and different demand during different periods, has become very costly), as well as in some cases, the fear of terrorist attack on the grid network, made the world leaders stop and think about one of the biggest necessities of the human race.

2.2.1 Smart Grid 1.0 – Smart Meters

We can say that the smart meters are the first step towards a smarter grid, the so called Smart Grid 1.0 [4]. This is more or less the stage where we are now: smart meters are implemented in houses and businesses, collecting data about their electricity patterns of usage, without the need for human interaction with the device as it was before, when there was a need for an utility company employee to come and read the numbers manually.

Invented in 1972 by Theodore Paraskevakos, this technology allowed for utilities to calculate the amount of electricity spent by the consumers, in between billing periods, an amount, as stated before, that had to be physically read every month, and then charge the customer. With the emergence of the smart meters, every house meter can automatically send to the utility the exact amount of electricity spent on an hourly basis, rather than the need for the utility to roughly guess the consumption as it was before. This meant knowledge of electricity demand on an hourly bases, allowing them to analyze this data and meet better the daily changes in demand, thus increase customer satisfaction and decrease overhead costs. In Figure 2 below we can see the difference in capabilities between the conventional meter and smart meter. While the conventional way of metering requires physical collection of usage data, either by collecting it manually of with automatic meter reading devices, which even though allow for faster data collection, require a physical presence. This data is then processed and stored in the database. With the usage of smart meters we have a slightly different process. The data is automatically sent on an hourly bases from the home area network, to which the smart meter is connected, through a NAN or WAN via a dedicated gateway in order to be stored in the database. This data can then be used by energy suppliers, distribution network operators and other emerging service companies.

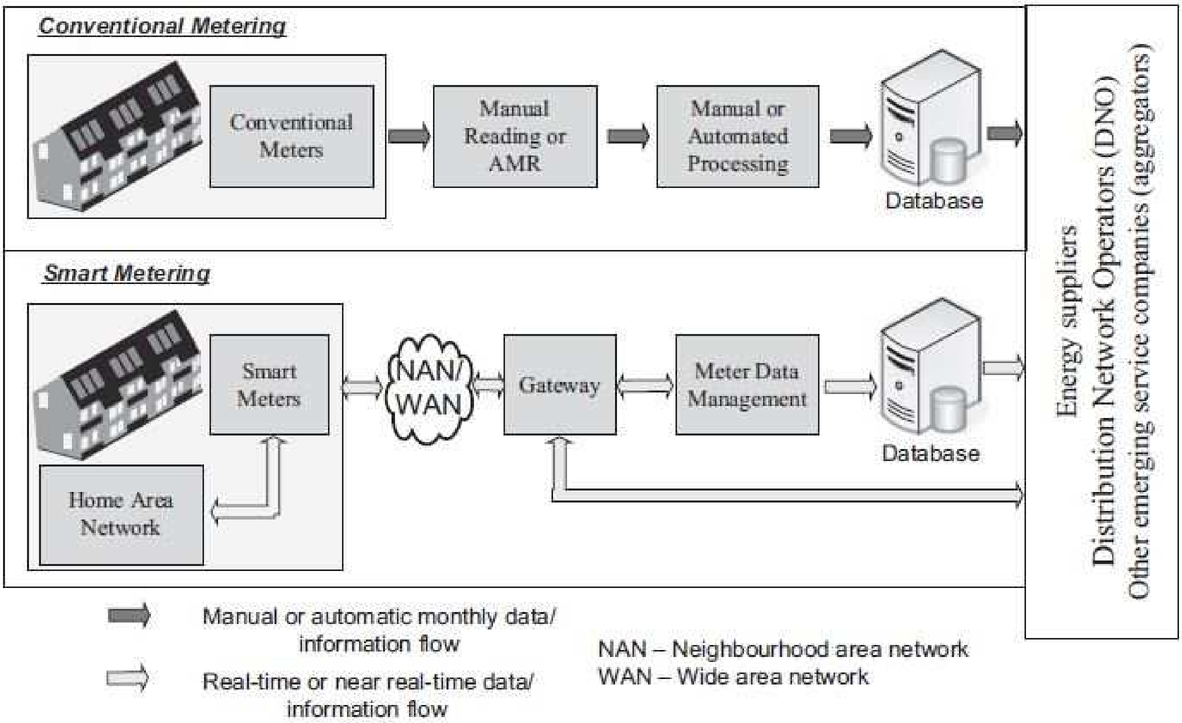

Figure 2 Conventional and smart meter

But this is only adding a new two way communication layer between the utility and its customers. The main purpose of collecting meter data is usually stated, or assumed to be, electricity management (such as demand side load management, efficiency consulting, energy savings, customer feedback/display etc.). More likely, data collection occurs simply because it can be easily and cheaply done by remote metering technology originally deployed to reduce meter-reading costs. This means fewer costs for utilities (since there is no need for hiring employees to measure data remotely as well as the lack of need of maintaining a fleet of vehicles to reach those meters remotely.)

So far, smart meters have failed to deliver smart grid benefits from a technical point of view: the network generally does not provide a full two way communication; the data displayed to the customer is usually stale data on a third-party web site, thus making any real-time pricing information obsolete; meters are not equipped to implement demand response load control.

Meter networks generally are not true two way communication networks – they are intended for polling meters and not designed to handle in-bound signaling for demand response strategies or to communicate with home automation systems, in-house devices or smart appliances. Even if meter networks were able to do so, the back software to support such applications is not yet available, or is in primitive state of development and not standardized. These networks do not provide a fully functional open premises information gateway to homes. Even if the meters did provide a gateway function, they would likely be implementing a top-down centralized control strategy, which in turn is the total opposite of distributed generation with the usage of renewables that the concept of smart grid puts as the main pillar of its existence.

As stated in the US Federal energy regulatory commission 2013 staff report [5], a utility launched a project in 2012 which allowed customers to automatically send their energy usage data directly to third-party providers, via their smart meters, which in turn allowed customers to download a smartphone application in order access data about their usage during the previous day. It also allowed them to learn about the impact of their energy use, possibility to extract data and make more informed decisions while cutting costs as well as receiving tips for saving money from their energy bill.

With this increase of information available on the usage amounts and patterns provided by smart meters, there is an increase in services that utilities have developed for customers, such as energy usage reports, mobile software applications which enhances and eases communications between utilities and customers, as well as energy management software intended to help users cut costs. Some utilities have decided to hire third-party software providers to help them develop these applications that help transform raw data into knowledge helpful for the customer. These applications can include information to users on how to cut costs by changing usage patterns, lower usage of certain high consumption appliances, and alternative rate programs.

Still, all these options provided by third-party providers, albeit useful for users, do not provide them with the information to make instant decisions from real time prices, let alone allow for machine to machine communication for automatic smart decisions. The availability of pricing and usage information at an instantaneous rate is essential in order to be able to implement incentivized demand response and load control schemes.

In the US, even though smart meters have been installed in many households, the benefits are yet to come. The state regulators [6] are pressuring them to provide the benefits they were intended to provide in the first place, which in turn will allow for better customer relationships, grid reliability (lowering possibilities of power outage), energy theft and bill accuracy.

2.2.2 Smart Grid 2.0 – Real time pricing and Demand response

This is the next stage after Smart Grid 1.0 and smart meters that do not comply with their intended usage. This means getting the smart meters to do what they were intended to do – provide real-time pricing, which will allow for demand response schemes and help smooth peaks of energy demand at certain times of the day and week. This in turn will allow users to plan their usage better, cut costs and help make the network more reliable due to flattening peak times and thus reducing the possibility of power outages.

Real time pricing

This scheme for electricity pricing implies giving information to customers about the cost of electricity prices at any given moment, either via their software or by partnering with a third-party software provider. This information can be distributed via text messages, email or a dedicated web site. The electricity prices usually change every 15 minutes in response to changes in demand, supply and market conditions. The main goal of this scheme is allowing users higher control of what they pay for at the end of the month, making them aware of how much they pay for what amount of electricity at any given moment.

Compared to the flat pricing model, where the price of electricity is constant over time and state regulated, during off-peak times, when electricity usage is low, the real-time pricing based scheme provides electricity with considerable lower prices. On the other hand, during peak periods, this price tends to be significantly higher. The high difference in price encourages users so change their usage behavior and incentivizes them to lower consumption during peak periods and delay some tasks to be executed during off-peak periods.

Some platforms allow for prices to be predicted a day before, while tracking daily changes of electricity price and allowing the user to make decisions ahead based on this approximation. This way, users can cut costs and plan their electricity usage behavior ahead. As we can see in Figure 3 below, prices that are predicted a day before are not exactly the same as in real time, a difference due to changing market conditions. This pricing scheme allows for prices to change every hour.

Figure 3 Real time pricing scheme with a day ahead prediction of electricity prices

Image source: https://rrtp.comed.com/live-prices/

Demand Response (DR)

The US Federal Energy Regulatory Commission defines Demand Response as: *“Changes in electric usage by demand- side resources from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized”*.

DR allows for demand side management by urging users to reduce their usage at times of high demands or encouraging them to increase consumption at off-peak times, either by setting different electricity prices for different periods or by using government incentives. This management technique has proved itself more cost effective for outages prevention than increasing electricity generation when demand surpasses supply. This is because it is more expensive to generate more electricity at peak times by turning on more costly electricity plants, than to incentivize users to reduce their demands at set times. Another way to use demand response is be encouraging users to use more electricity at low usage times, thus flattening the electricity demand curve as much as possible. This encouragement also saves electricity generation costs, since coal plants (which are the main source of electricity at the moment) are most cost effective if they work at high, almost full capacity. Even if the electricity was generated from a renewable source, such as PV panels, there would be an encouragement for users to use more electricity at peak sun hours, and less during the night, when the generation from this source would be zero.

The electricity supply is with an exponential growth during one day, since it takes time for the generator to start and then it reaches its optimum electricity production. The demand on the other side is somewhat variable (see Figure 4), with peak demand during the afternoon. As we can see in Figure 5, depending on the electricity demand (D1 or D2), we can have different prices (P1 or P2) which are set for different quantities of electricity (Q1 or Q2). By using lower amount of electricity (Q2), the price changes by ΔP.

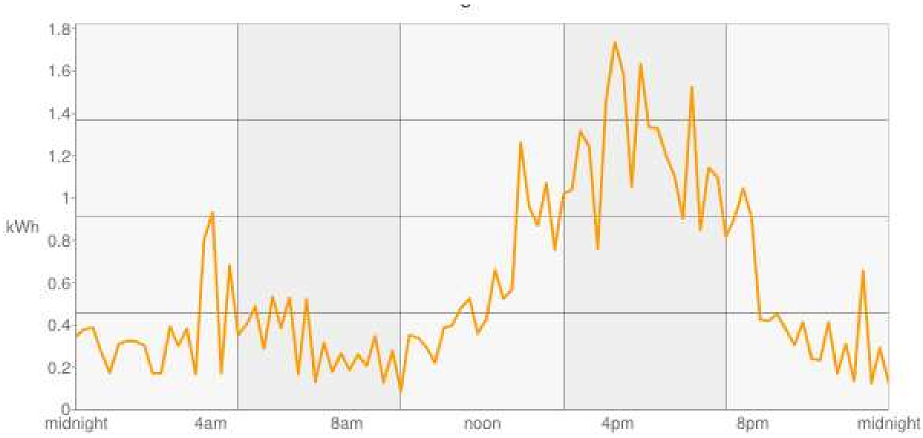

Figure 4 Daily usage of electricity

Image source: http://www.mytruecost.com/news/2013/10/1/daily-email-with-your-smart-meter-data

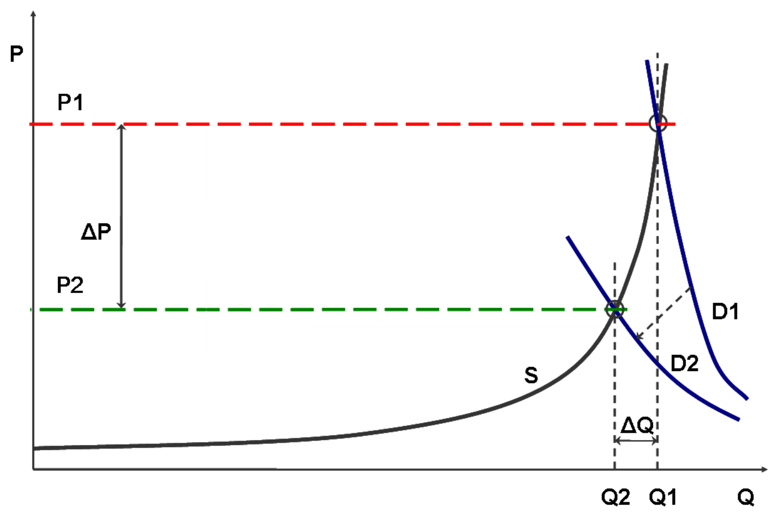

Figure 5 Electricity demand and supply and how it affects prices

Image source: http://publish.illinois.edu/incentive-pricing/

Demand Response benefits

Demand response has many benefits for both sides of the electricity market. Some of them are [8]:

- Financial cost cutting – both on the generation and consumption end. From the generation point, when high demand periods are smoothened, there is no need for them to turn on more generation plants which are very costly by operating only during these times. From the customers point, they save money by not paying the higher price charged during high demand periods. They user load scheduling or on site generation in order to relieve load from the grid. The energy saved is called negawatt power [7], representing a theoretical amount of energy saved.
- Grid reliability – with shaving peak demand, the grid becomes more reliable and stable by avoiding blackouts and equipment damage due to unsynchronized equipment and change in frequency and voltage. This allows for the power grid to accommodate more requests without the need to build more plants and/or turn on plants that generate higher costs (gas, hydro). It also allows for renewable energy sources, such as wind and solar, to be implemented in the grid more easily, since generation from those sources is unreliable itself.

Types of Demand Response

Demand response can be achieved by using one of these two demand managing techniques: load shedding and direct load control. We will elaborate them to some extent in the following section.

a) *Load shedding (scheduling)*

One of the ways to respond and manage peak demand is by scheduling loads. This means turning of devices at peak times, devices which do not reduce productivity and are easy to turn on/off. If the load they are producing is non-critical, the device can be turned off for a certain period of time, i.e. during the time of peak demand. This helps flatten the demand and lower the chances for outages that can lead to blackouts, something that happens when the generator is not capable of producing the needed electricity to satisfy all customers. Figure 6 shows how the load is flattened by delaying some tasks. Usually generating utilities make deals with high consumption industrial customers to turn off some of their equipment at high peak demand. Moreover, some large industrial consumers have their own generators on site, so when demand is high, utilities urge them to use their own generators, thus reducing overall demand, without reducing their own. In Figure 7 we can see that the load is only delayed, not lowered overall.

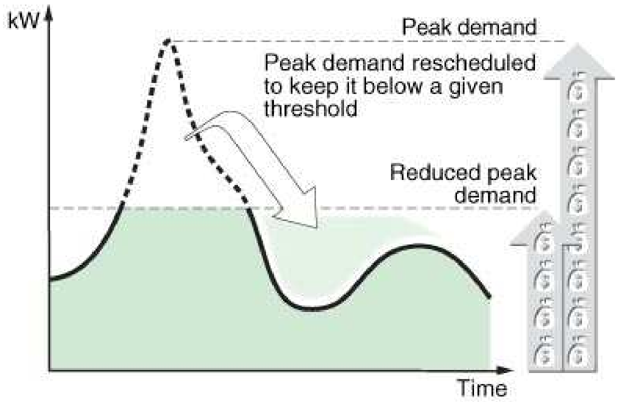

Figure 6 Load scheduling management

Image source:

http://www.electrical-installation.org/enwiki/Energy_saving_opportunities_-_Load_management

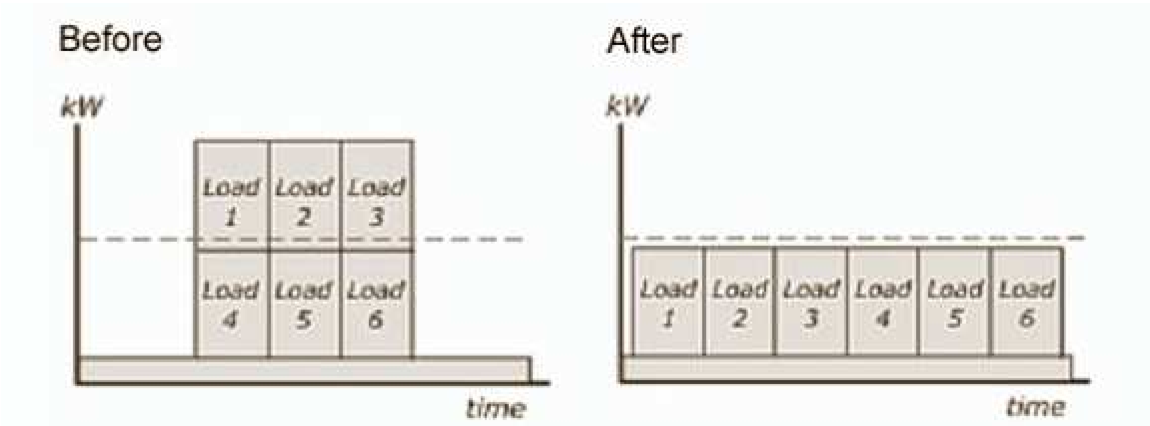

Figure 7 Load scheduling for demand peak shaving

Image source: Phillip Yeung, *Reducing energy costs with peak shaving in industrial environments*, September 2007

Another method of load shedding is with the so called *rolling blackout.* Rolling blackouts are intentional outages of some parts of the electricity network, done by the utility companies to prevent large outages at high demand times. this is done during different periods at different parts of the grid. As a technique it is mostly used in developing countries, since in developed countries, electricity delivery is well managed and planned ahead in order to avoid discrepancies between electricity supply and demand. This is now further enhanced with the data used from the smart metering devices.

b) Direct Load Control

The second demand management technique is direct load control [9], which is based on a contract between the power utility company and its customers. According to this contract [10], the company may control the residential appliances’ operation and energy consumption remotely during peak hours in order to balance the grid’s power consumption levels. DLC poses privacy concerns for users, however, and it is questionable whether customers would give such control to the company willingly. Still customers are incentivized to use this method by receiving tariff discount in exchange for the discomfort they might feel.

Some of the possible options for the privacy concerns is giving access to control only few high power and easily delayed appliances; which will require labeling appliances in regard of these characteristics. This policy will offer customers a choice between using and not using DLC, by offering them some incentive in order to receive permission to apply it.

Before we can create DLC plans by labeling devices according to their power consumption and operation urgency, we must consider whether a device’s operation can be delayed or not. A *non-schedulable device*, such as a light bulb, TV or a PC needs to be operable at any time the customer wants since they are essential for the quality of living. On the other hand, a *schedulable device* can be planned to start operating at a future time. Schedulable appliances can be further categorized by whether their operation can be paused or not. An *interruptible device*, such as an air-conditioner, heater or EV battery, can be paused and re-activated later. *Non-interruptible devices*, such as washing machine and dish washers, need to be operable continuously once started.

When labeling devices, it should provide information about power consumption, how much time does the device operate, what is the preference of the utility company – which appliance do they prefer turning off? According to these labels, devices can be further classified in different power usage groups. So at peak times, when a load request comes to the utility company, they can assess the appliance type according to the label and decide whether it will be turned on or it will be put in a queue for latent operation. So the request with the lowest power demand and primacy will be served first. The algorithm also allows for appliances labeled non-interruptible to run continuously once turned on. When an operation is over, the appliance is put back in a queue and assigned lower priority than appliances not yet served.

Let's say a utility company offers a pricing menu for getting access to an AC and change the temperature, and p(u;a) = ṗ - au, where $\dot{p}$ is the regular fixed price, a_{nxn} = diag (a_1,a_2,….,a_n) is the discount matrix specifying discount parameters a_i for each group i, u_{nx1} is the controller of temperature and n is the number of groups under control[11]. When the AC is cooling, the utility company will raise the temperature at peak times to reduce the load, i.e. set the controller at u>0, which in turn gives customers discount rate p(u;a), otherwise, they will pay the regular rate ṗ.

This type of DLC is closely related to thermal comfort. Comfort is a function of the following parameters: a, which is a physical value and μ, which is a threshold value, and two time thresholds – $T_{min}(a,\mu)$ – minimum time required to reduce a hypothetical discomfort metric, a, to the wanted value; and $T_{max}(a,\mu)$ – time needed for the discomfort metric a to reach a maximum threshold, μ [12]. The objective of the utility while using DLC is to keep the discomfort level below threshold (see Figure 8).

$$P_p(t) = \sum_{j=1}^{N<M} \left[P_j(t) \right] \leq B_p(T), \forall t$$

Figure 8 Pp is the total load, Bp is the total capacity, M is the number of appliances and Pj is the power of appliance j

Advanced Metering Infrastructure (AMI)

Advanced metering systems are comprised of state-of-the-art electronic/digital hardware and software, which combine interval data measurement with continuously available remote communications [13]. These types of systems allow the frequent measurement of time based and detailed information to different parties. The whole infrastructure is comprised of a measurement and collection device at the customer site, a communication network between the utility and the customer, as well as management systems and data processors at the utility site. We can see the AMI in Figure 9 on one side we have data collection (via electric, water or gas meter) at the customer site. On the other side we have the utility/third party data reception and management which is made of an AMI host that forwards data to a meter data management system. These two sides are connected via a data transmission network (broadband power line, radio frequency, public networks).

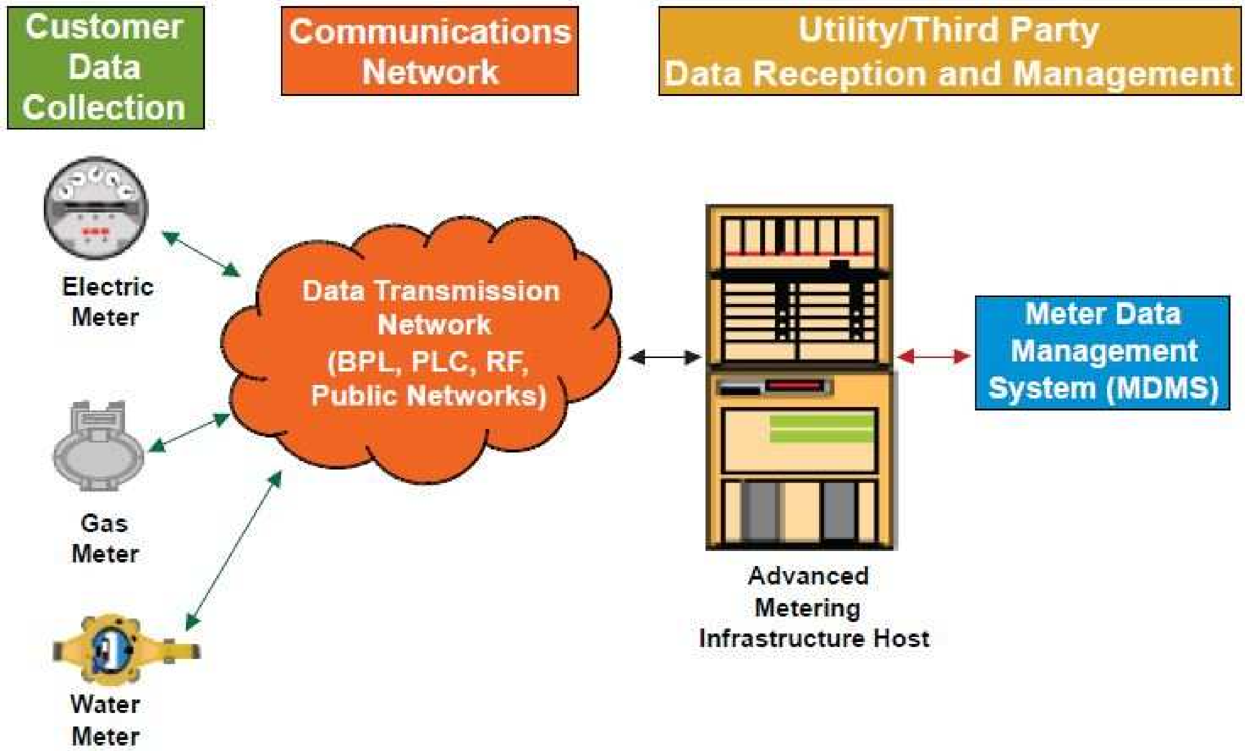

Figure 9 Advanced Metering Infrastructure

Image source: https://www.ferc.gov/EventCalendar/Files/20070423091846-EPRI%20-%20Advanced%20Metering.pdf

AMI Benefits

A broad categorization of benefits that utilities and customers enjoy by using AMI is:

- System operation efficiency – by using AMI, there is no need to read meters physically anymore, thus reducing the need for manual data collection, entry and processing. It also increases data accuracy, easier energy theft detection and better outage management.
- Improved customer service – no need to guess the amount of electricity spent monthly, early detection of failure, providing customers with different pricing schemes, possibility for DR management techniques, and offering real-time based prices which gives more control to the customer, thus saving them money.

- Cutting costs – the utility saves by reducing equipment and maintenance cost, faster solution of outages as well as no costs connected to vehicles and people reading meters manually.

Types of appliance load

Demand response techniques and whether they are applicable, depend heavily on the type of loads they are to model. Different types of loads are elaborated in the following section.

a) *Standard load* – these types of loads are closely connected to the quality of living and are essential for the operation of the household. Devices like lights and TV/PC are easily turned on/off, usually increase the overall load by adding additional flat power levels and belong to this category of loads. Although easy to turn on/off, their load is related to the time of usage, hence usually require power in the afternoon and night. Standard loads cannot be controlled by direct load control, but the load scheduling technique can be used to incentivize users to lower this type of load during peak demand time.

b) *Flexible load* – these types of loads are usually connected to heating and cooling devices (AC, refrigerator). They are closely related to the physical comfort index, which represents the margin until which point the load can be reduced without disturbing the comfort. This threshold is explained in the DLC section. The operation of the AC can be lowered until this discomfort threshold is met. For the refrigerator, it is the temperature that still keeps the food cold.

    - *Refrigerator operation* – this appliance has a cycle of function which allows it to turn on/off by itself. In an hour long cycle, the fridge goes off and then again on. It is with a constant load for half an hour and then goes off again. When the next cycle starts, the fridge is off for more than an hour, and the compressor turns on (which happens because the temperature has risen to the maximum threshold), producing a noticeable power spike followed by constant load. We can see the fridge operation in Figure 10. The consumption in a cycle is approximately 83 Wh, while the daily load of this device is 2kW. The footprint of the fridge is considered to be *periodic*, since the on/off cycles of the operation are almost the same. In Figure 11 we can see a separate operation cycle.

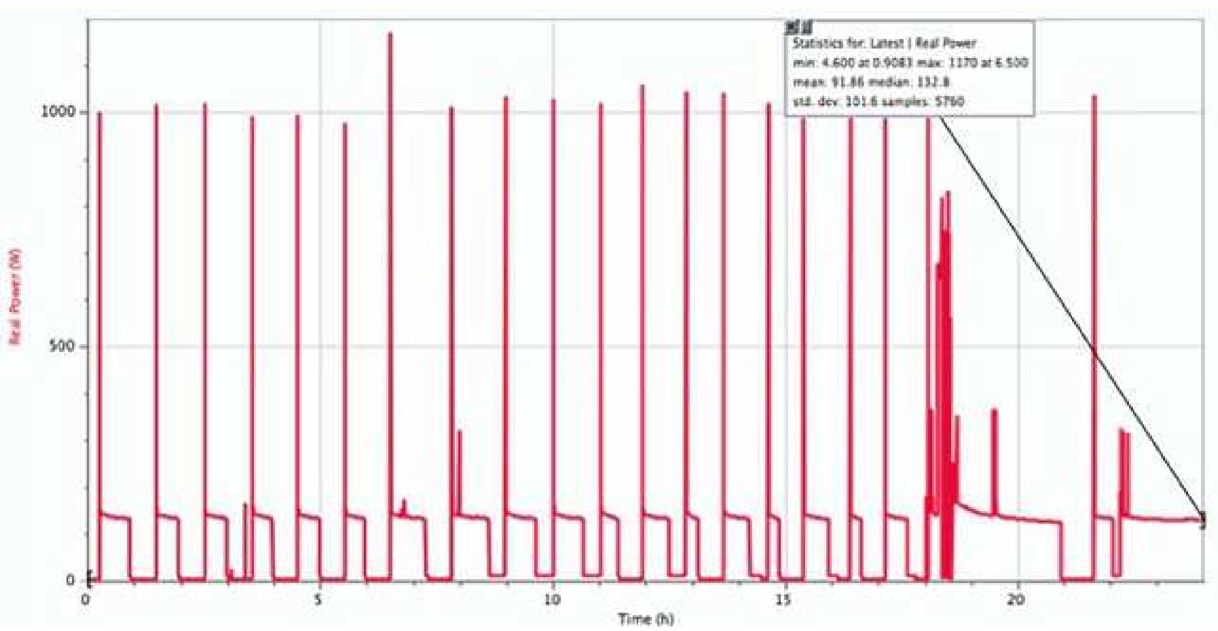

Figure 10 Fridge operation

Image source: http://www.wired.com/2011/11/power-and-electric-motors/

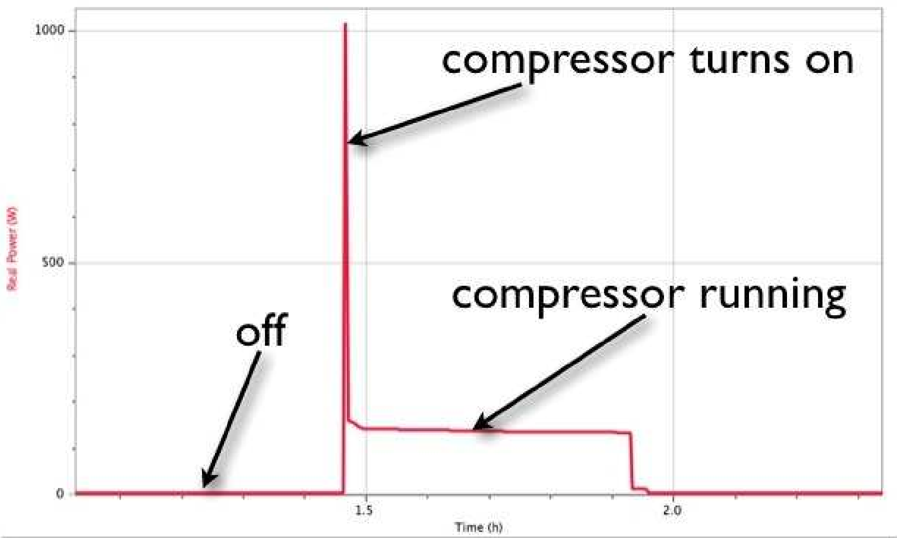

Figure 11 Operation of one fridge cycle

Image source: http://www.wired.com/2011/11/power-and-electric-motors/

c) *Elastic load* – these types of loads are related to tasks that can be delayed such as washing machines, dish washers and electric vehicles. There is also a comfort threshold connected to elastic loads, but unlike thermal thresholds as in flexible loads, this comfort threshold is related to time, thus providing a certain deadline until when the task has to be finished. The duration of the load is crucial in setting the maximum starting point of the load. In Figure 12 we can see different tasks with different loads and durations scheduled to start at different times, which are before the deadline set. Here *d* is the deadline value, *a* is the task initiation point, *s* is the duration of the task and *p* is the power needed to execute th task.

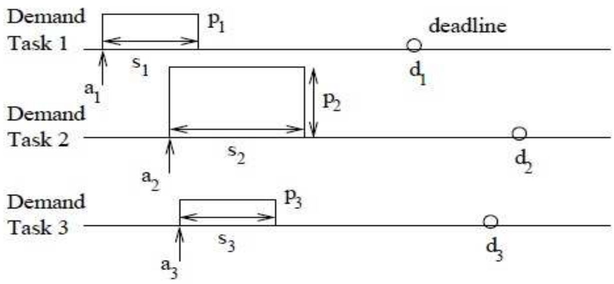

Figure 12 Comfort level of elastic loads connected to task deadline

Image source: G.Koutitas, *Lecture notes on Green ICT*, International Hellenic University, 2013/14

A typical elastic load is considered the operation of a washing machine. This appliance operates a few days in the month for only a couple of hours, making it easy to be delayed and used when electricity demand is overall low. In Figure 13 we can see its operation. The different spikes and durations of the load is due to the different operation states of the machine, such as washing, rinsing, spinning etc.

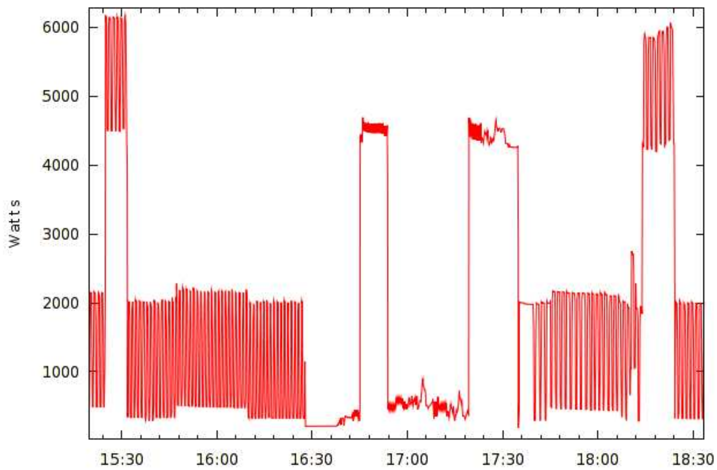

Figure 13 Operation of a washing machine

Image source: http://www.stahlke.org/dan/powermeter/

This type of operation is considered as multistate, as it has different electricity demanding states in one cycle of operation. In this example, the highest spike is 6kWh, while the whole operation electricity demand is 3kWh.

There are two ways to measure different household loads – intrusive or non-intrusive load monitoring.

- Intrusive load monitoring – this is achieved by putting a monitoring device on each appliance. The results achieved with this method are more accurate and the only inaccuracies are the ones from the metering device.
- Non-intrusive appliance load monitoring (NIALM) – this method includes one meter that monitors the load of all household appliances for a period of time. After that the load is disaggregated and appointed to different types of appliances, using these steps: event detection, clustering and classification. By event detection we can see when the appliance load started and its duration. After that we cluster the similar appliances into groups. Then we classify each group to a type of appliance. In Figure 14 we can see the disaggregated loads of several household appliances.

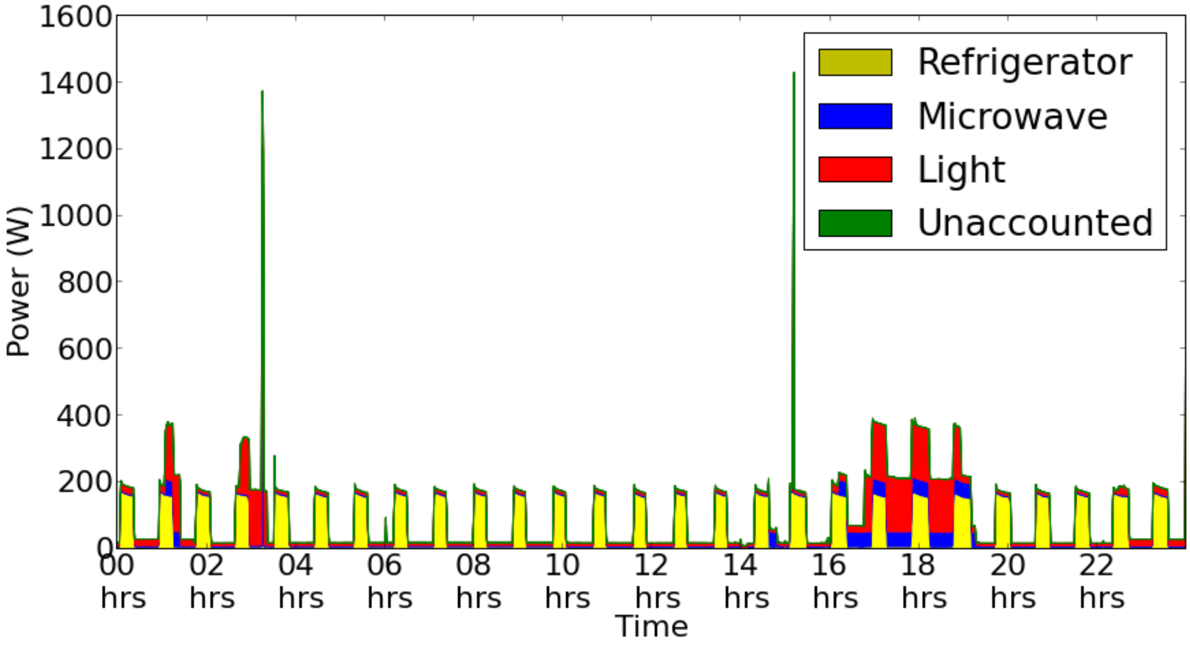

Figure 14 Disaggregated loads by using NIALM technique

Image source: Batra N., Dutta H., Singh A., *INDiC: Improved Non-intrusive Load Monitoring Using Load Division and Calibration*, IEEE, December 2013

2.2.3 Prosumer electricity production

The most popular and developed ways for households and industrial customers to produce electricity on their own, whether in order to reduce demand at peak times or go totally off the grid, are solar and wind energy production. Using these renewable sources of energy generation is a more independent, eco-friendly way of producing electricity. Solar panels or small wind parks are installed closely to the consumption point in order to reduce distribution costs.

Solar photovoltaic electricity generation systems

Depending on the number of solar panels in an array and their size, different systems are capable of producing different amounts of electricity. As we can see in Figure 15, the curve of the PV generation is highest during the day and lowest and non-existent during the night. The generation of electricity also depends on the peak sun hours (PHS) the area they are installed at has. We can see a comparison of production between two PV systems: one has a capacity of 1.5kW with a peak of 1kW and a total of 7.3kWh daily, and the other has a capacity of 3kW with a peak of 2.1kW and a total of 14.5kWh daily. Generation for this area, i.e. sunrise starts at 7am and stops at 7:30 pm. We have to take into consideration that generation from solar panels is different in different periods of the day and in different months of the year. The average daily consumption for this particular household is 20.5kW with a peak of 1.2kWh in the evening hours. Since the generation and demand don't meet at certain points, there is a need for storage batteries in order to have electricity during the night as well. Also as we can see in both cases daily demand surpasses the generation capabilities, so in order to cover the demand there is a need of at least a 4.5kW system which produces 21.9kWh daily. Another solution is to lower demand to at least the generation point of the PV system. Of course this is only a calculation for this particular area for this amount of PSH. The surplus of electricity, if there is any, can be sold back to the grid. This so called *Solar feed-in tariff* is what the grid will pay the PV producer for the electricity he exports back to the grid.

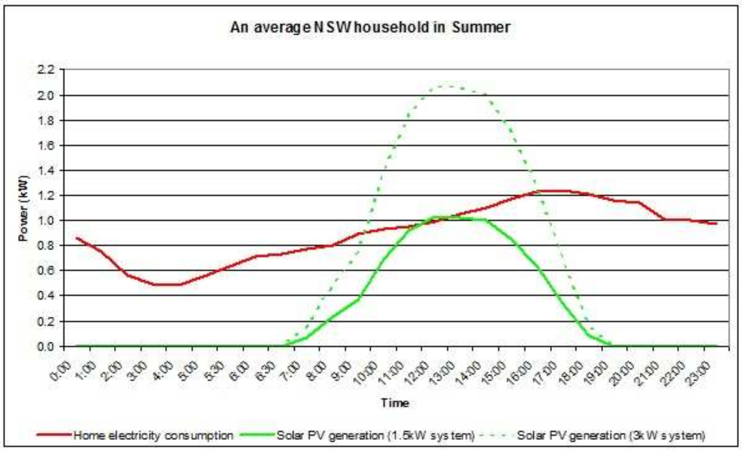

Figure 15 A graph plotting the electricity consumption of a household and the production capabilities of two different PV systems.

Image source: http://www.solarchoice.net.au/blog/home-energy-consumption-versus-solar-pv-generation/

Wind electricity generation

Wind parks are made of a group of wind turbines at the same location that produce electricity. Also, small wind turbines can be mounted on a house for lower generation needs such as a single household. The generation of electricity from this renewable source depends on the wind speed and frequency. For a wind turbine to start working there is a need of 3.5m/s speed or more. Usually they are installed on top of houses or on higher places in order to generate more electricity. There is more wind usually during the night, so it is a good complement of PV panels. In Figure 16 we can see the general output of a wind turbine. The *cut in speed*, i.e. the speed needed for any electricity to be produced is 3.5m/s. The *rated output speed* is the speed between 12 and 17m/s and what allows the wind turbine to operate at its maximum capacity. The power generated at the maximum capacity is called the *rated power output*. The turbines are made in such a manner, that when the wind reaches a certain speed, the rotor stops working as to avoid equipment damage. This is called a *cut-out speed* and is usually when the wind reaches a speed of over 25m/s.

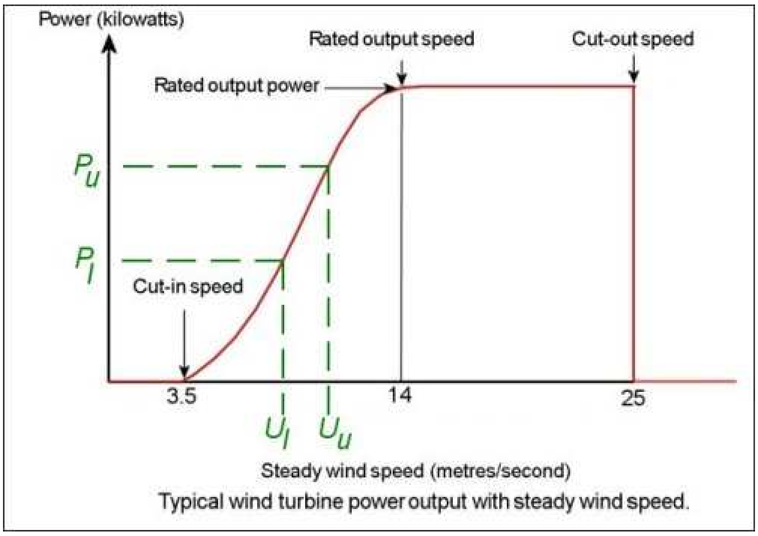

Figure 16 Wind turbine power output

Image source: http://www.wind-power-program.com/turbine_characteristics.htm

Since the power output of a wind turbine is a variable dependent on the weather conditions, it is very hard to predict the power output for a particular time. This is the reason why in our model we only use PV systems for electricity generation.

2.3 The entities of Smart Grid 3.0

When transitioning to smart grid 3.0, there is a need for the basic new era entities to be present. These entities, such as prosumers, microgrids distributed generation, virtual power plants and transactive energy markets are essential for a goal to be achieved, which is: distributed generation at the end of the grid, by employing renewable energy sources at the household vicinity, islanding generation for better outage control and giving users more power by allowing them to generate electricity for themselves, and also sell to other users or back to the power grid.

2.3.1 Microgrids

Microgrids are a group of localized generation or storage centers that operates either on its own or is connected to the central power system. These are usually dispersed as distributed generation centers closer to customers. Until now, the DGs are usually disconnected when there is an outage, in order not to damage the network. In some countries, DGs act as a backup power in times of outages at the local level [14]. As the microgrids act as a group of DGs, it can be seen as a miniature of a large interconnected grid that can provide the demanded power as well as go from islanded mode of operation to interconnected and vice versa [15].

Microgrids can operate in island more or import electricity from the grid if necessary. Generation resources are usually renewables, such as solar and wind, since they can be localized closer to loads and produce smaller amounts of electricity. Multiple distributed generation sources and the possibility to isolate the microgrid from the central power grid makes this network a highly reliable electricity provider. The microgrid is set to remain operational in an autonomous mode after islanding and meet the corresponding load requirements [16]. Implementation of microgrids is highly frequent in places where blackouts are common due to severe weather conditions and where the central power grid is subject to frequent cutouts.

Types of microgrids

We can further describe the types of microgrids as:

- Isolated microgrids – microgrids operating autonomously, i.e. not connected to the central power grid.
- Islandable microgrids – this type is fully interconnected and synchronized with the centralized power grid, and is capable of producing and consuming electricity to/from the central grid. It can maintain some level of electricity supply during outages.

In Figure 17 we can see how the microgrid is connected to the utility centers, distributed generation points and loads.

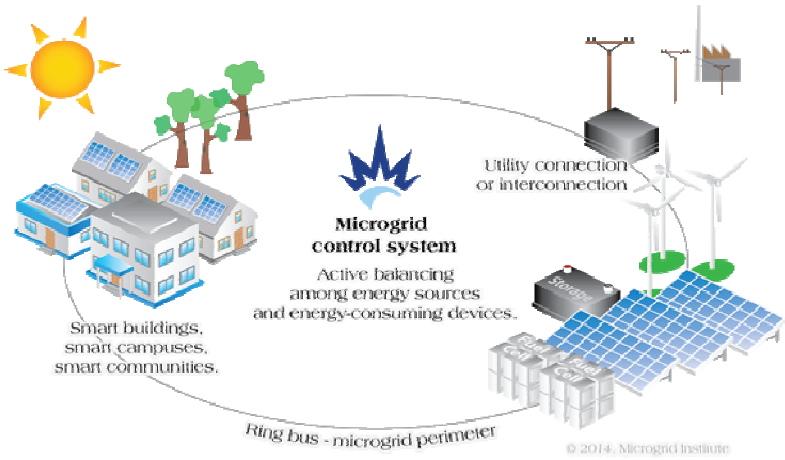

Figure 17 Microgrid control system

Image source: http://www.microgridinstitute.org/about-microgrids.html

Island mode – this is when the microgrid is not connected to the utility grid and operates on its own. This if only possible if the microgrid's produced capacity *P(t)* is equal or higher than the sum of the total consumption load *c(t)* of all the users *i* in the microgrid set *M*, as expressed in the following formula (Figure 18):

$$P(t) \geq \sum_{i \in M} c_i(t), \forall t$$

Figure 18 Island mode operation condition of microgrid

Microgrid benefits

Some of the major benefits when implementing distributed generation for microgrids are:

- Transmission and distribution costs – since the generation points are closer to the loads, distribution costs are significantly lower.
- Self-endurance – even if there is an outage, the microgrid users will still have power to function without the utility grid. Moreover, the microgrid can supply the central power grid with electricity until the outage is repaired.
- Lower carbon emission – since the energy sources usually used in microgrids are renewables, the amount of carbon footprint is significantly lower than from traditional power plants.

- Rural environments – this is a good solution for users who want to consume electricity, but cannot connect to the central power grid because it does not reach their location.

2.3.2 Virtual Power Plants (VPP)

Thought often used interchangeably with microgrids, VPPs are more connected to the control of generated power from sources that do not necessarily need to be in a proximate location. By aggregating many small distributed generators into a portfolio, they become visible to the system operator who can then control them remotely. The output from a set VPP is made to have similar characteristics as a regular generation unit. The portfolio of generators in a VPP is classified by the clustering of generators:

- Geographically proximate generators – the distributed generation centers are geographically close to each other, which may not be the best portfolio set. If the wind turbines are set far away from each other, electricity generation is bound to be more continuous, as wind blows at different times in different places. Similarly, the solar panels that are more dispersed, generate more electricity together due to changing weather conditions at different places.
- VPP size – the size can range from small to large depending on the number and size of distributed generation centers. The size of DG is usually small, but when pulled together, they can be considered as a big enough sources to compete with the central power grid. As the number and distance of DGs increases, the need for seamless control is higher which will facilitate communication and information exchange between system operators and DGs.

2.3.3 Prosumers

With the intention of becoming more sustainable, the power grid is heavily exploring ways of integrating distributed renewable energy generation points and storage units at many levels. Maybe the most obvious integration is at the generation level, where renewable sources will supplement and hopefully replace traditional power generation sources. Another integration point to be considered is at the distribution level where smaller scale renewables generation offer cheaper and more environment friendly options to customers. In recent years, an integration point located below the distribution level is emerging, allowing users to produce their own electricity [17]. This seams as the most logical option, since renewable energy generation is most effective when produced In small scale and close to the load.

Users that have installed distributed generation and energy storage centers, making them producers as well consumers, are called *prosumers* [18]. There are two modes of operation:

Prosumer – if $p(t)>c(t)$

Consumer – if $p(t)<c(t)$

where $p(t)$ is production over time and $c(t)$ is consumption.

Such generation points are needed with the emergence of the renewable energy sources. It allows users that do not have access to the central grid to generate electricity for themselves. Another possibility is for prosumers to produce electricity for themselves and sell the surplus back to the grid. This is regulated by the feed-in tariffs that are set by the utility company.

Examples of prosumers include a building with an EVs that provide storage services to utilities, households that produce electricity and exchange power among themselves or sell back to the grid and a microgrid that sells electricity to another microgrid or utility when demand is high.

One of the rising ideas is that it will become cheaper for users to produce electricity than to buy it from utilities [19]. The emergence of cheaper solar system equipment and abundance of it makes users more confident about producing their own electricity. There are some countries that encourage users to produce electricity and sell it back to the grid in order to earn some money. Other utilities install solar panels on users houses and then sell the electricity at a lower rate than before. There are many ways of going about the produced electricity, with one major issue still present: storage.

When Wes Kennedy started engineering solar systems in mid 90's he has one integration option – batteries [20]. Since the emergence of solar systems, the most important technology was storage related, since prosumers had to store electricity for later usage. With the emergence of policies requiring utilities to connect to prosumers, the need for storage has become marginalized. Still, if prosumers decide to operate off-grid, the need for storage devices is essential, be it batteries or EVs. In Figure 19 we can see a graph showing drastic drop in battery price in the years to come, which is encouragement for prosumers that want to operate off the grid.

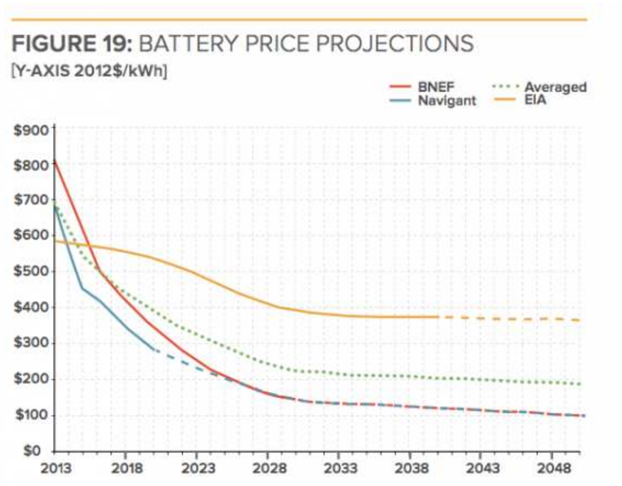

Figure 19 Battery price projections

Image source: http://www.greentechmedia.com/articles/read/where-and-when-customers-may-start-leaving-the-grid

The drop of commercial and residential solar is also promising, with a drop of around 60% in the recent years.

One of the key characteristics of prosumers is that they assume different roles at different times: at one moment they could be producers at other they are consumers. The control architecture based around prosumers is different than the one that operates now. The prosumers based architecture takes proactive and distributed control instead of waiting for centralized hierarchical control.

Traditionally, energy flow has been one way, in bulk from the generation point through transmission and distribution lines to the consumers. With the emergence of DG and prosumers, the need for a different design that allows for electricity to flow bi-directionally at distribution level is essential. Prosumers are economically motivated to produce and store electricity, to operate a power grid and transport electricity, and be motivated to change usage behavior in order to sell excess to utility companies or residential and commercial users. We can see the concept of a prosumer in Figure 20, which can represent a household, a certain device powering on solar charger, a utility company, industrial facility or a commercial building. Prosumers can produce, store, buy or sell electricity as needed.

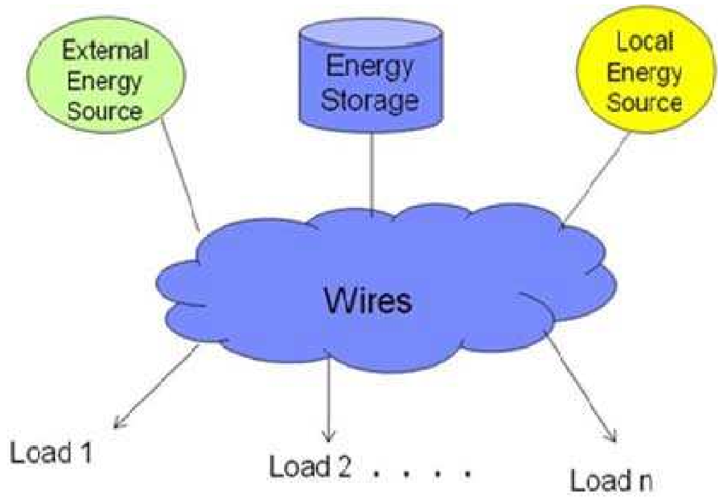

Figure 20 Concept of prosumer

Image source:

http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=5759167&url=http%3A%2F%2Fieeexplore.ieee.org%2Fxpls%2Fabs_all.jsp%3Farnumber%3D5759167

Prosumer based architecture

In order for prosumers to interact among each other or with consumers or utilities, there is a need for a layered architecture [21]. Each of the control layers provides a more intelligent mechanism for grid control. In Figure 21 we can see the prosumer based architecture, made of 4 layers.

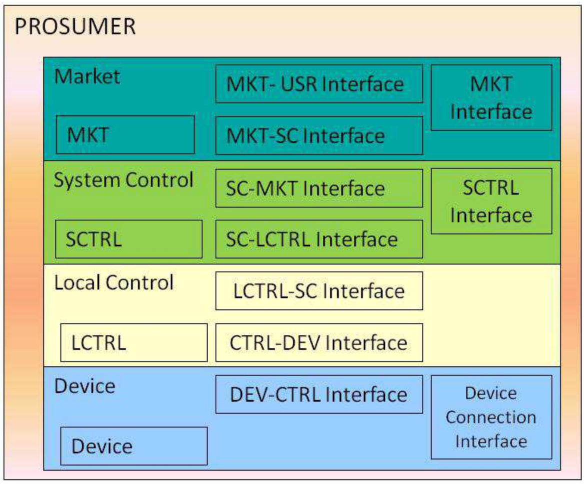

Figure 21 Prosumer based architecture

Image source:

http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=5759167&url=http%3A%2F%2Fieeexplore.ieee.org%2Fxpls%2Fabs_all.jsp%3Farnumber%3D5759167

a) Device layer – represents the physical connectivity of electric components.

b) Local control layer – the control mechanism of devices such as the battery charger of an EV. It can include electromechanical, power electronic and software components. It is capable of acting based on local information and must provide interfaces in order to interact with the system control layer.

c) System control layer – provides the control needed to meet the functional and performance system level objectives, such as volt-var regulation, economic and secure operation, system restoration, loss minimization etc. It uses applications like state estimators in order to monitor system state. The EMS and DMS applications of electric utilities are an example of system control layer.

d) Market layer – this layer decisions are updated by the previous layer according to system constraints. It uses this information and uses advanced economic and financial applications such as location marginal pricing calculation, risk management, load and price forecasting etc., in order to generate control actions for the layers below it or price signals for the electricity users.

2.3.4 Transactive energy

Transactive energy is defined as "a means of using economic signals or incentives to engage all the intelligent devices in the power grid – from the consumer to the transmission system – to get a more optimal allocation of resources and engage demand in ways we haven't been able to before"[22]. As stated before, this translates into the distribution system becoming a bi-directional system connecting resources like DR and household prosumers and allowing load management. In Figure 22 we can see the transactive energy market, where microgrids can be connected to the central grid, residential customers can produce and store electricity, EVs can be connected to the retail market, and all this for the sole purpose of electricity exchange and storage.

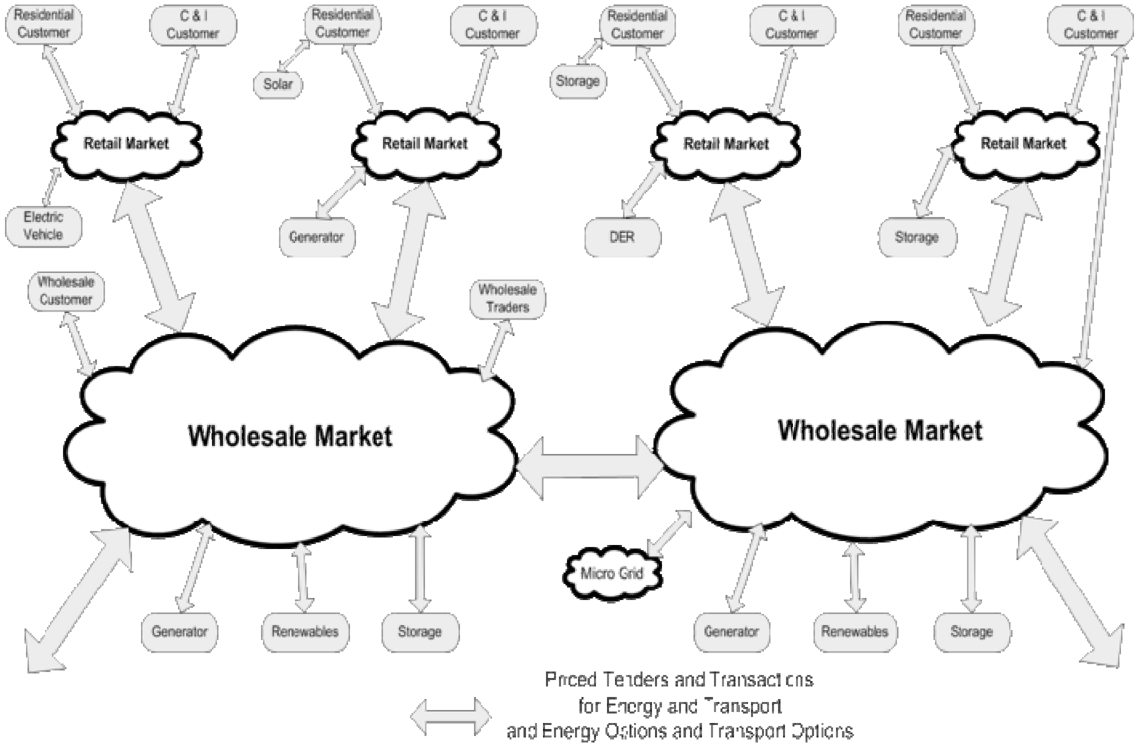

Figure 22 Transactive energy markets

Image source: http://www.pointview.com/data/files/2/1062/1878.pdf

The TeMIX protocol developed enables high speed, high volume information exchange for electric energy transactions [23]. The protocol developed facilitates the process of negotiation, contracting and delivery of electricity between parties. Generators and consumers can both take the roles of buyers or sellers on the market. Figure 23 shows how different parties such as generators, traders, customers, retailers, transmission and distribution providers, can transact via the TeMIX protocol network. Communications involve a series of priced offers that lead to a transaction.

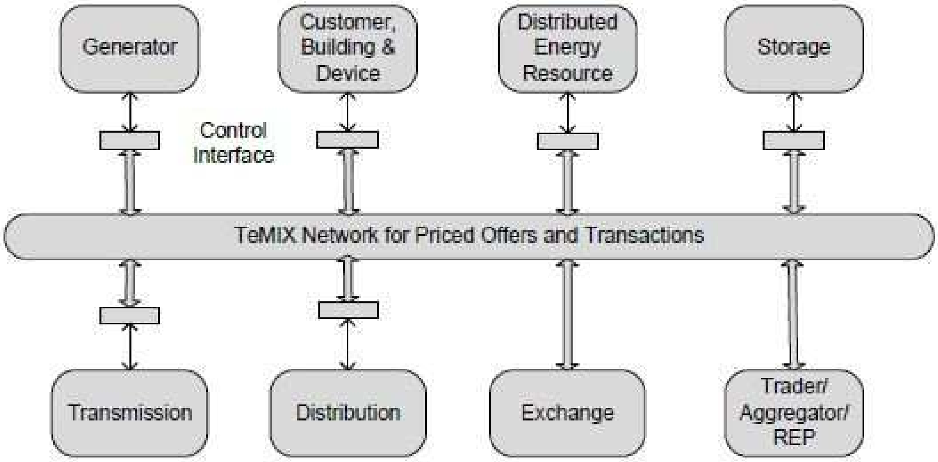

Figure 23 TeMIX network

Image source: http://www.pointview.com/data/files/2/1062/1878.pdf

TeMIX operations

The protocol described before allows for decentralized decisions and coordination of price offerings among different parties. Transactive operations for different players are different and are explained below.

a) *End device transactive operations* – each device that produces, consumers or stores energy (renewable energy generator, home appliance, EV) is considered an end device. In Figure 24 we can see the operation of these end devices. The control interface has three functions: to determine the device's optimal operating level, receive and make optimal priced offers and enter into optimal transactions.

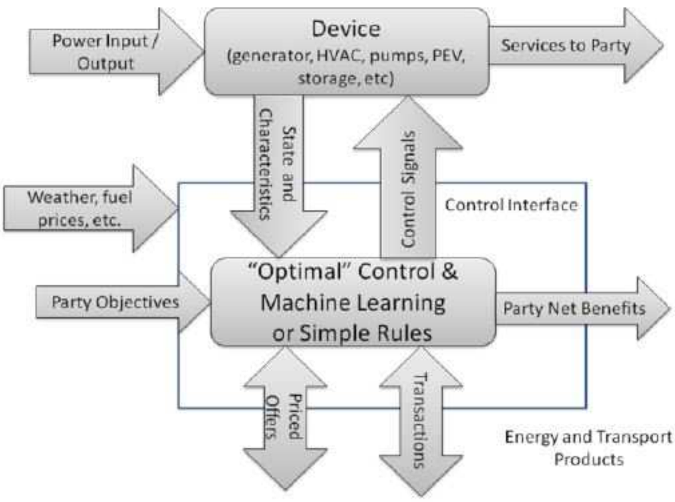

Figure 24 End device transactive operations

b) *Transport transactive operations* – companies that provide transmission and distribution services fall into this category. In Figure 25 it is shown the transport transactive operations, where we can see that the primary of optimization for transport products is to determine the priced offers to other parties and determine which offers to accept. A transport operator can also choose to buy electricity at one location and sell at other location, as well as offer a price to transport electricity between two locations.

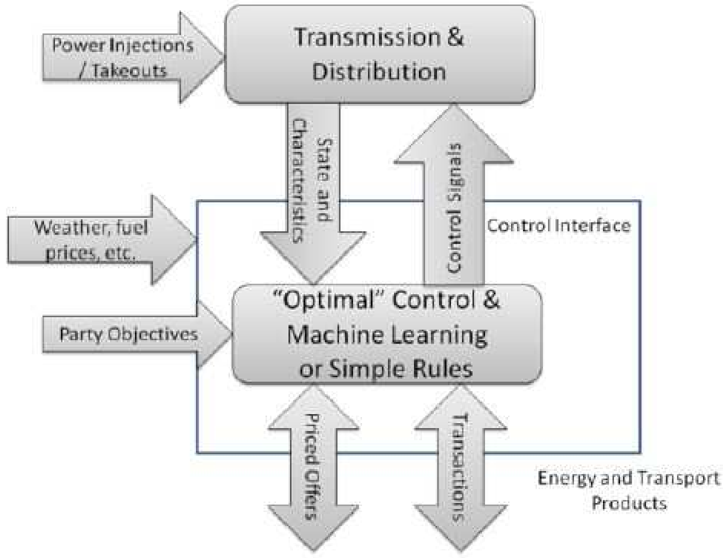

Figure 25 Transport transactive operations

c) *Intermediary transactive operations* – parties such as brokers, marketeers, retailers that have portfolios of energy and transport products for third parties fall into this category. Figure 26 shows the intermediaries transactive operations, which includes intermediary services between buyers and sellers.

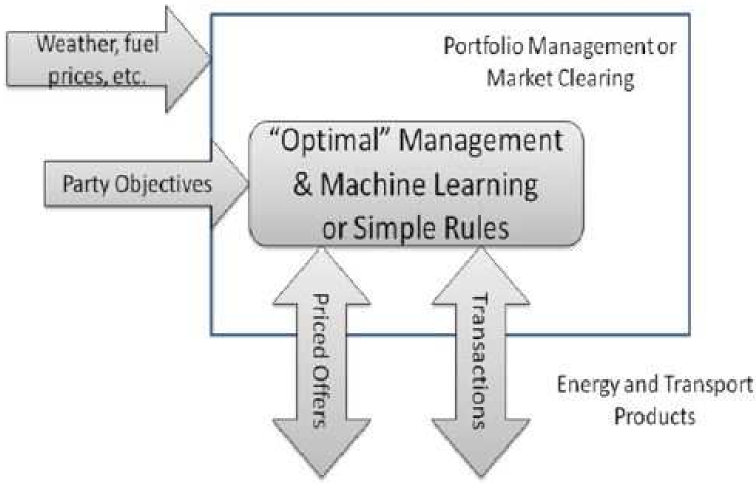

Figure 26 Intermediary transactive operations

2.4 The energy market

Energy markets are places where interested parties meet to trade with different kinds of energy. The energy market had a traditional way of generation and distribution, usually there was one company, private or public, and that generates and distributes energy to the end users. The government gave a monopoly franchise to this company, allowing it to be vertically integrated and be responsible for the whole process, from beginning to end. The process of generating and delivering electricity to the end users is the following [24]:

a) *Generation* (power plants and generators) – the production of the electric energy by transforming other forms of energy, be it coal, atomic fusion, wind or solar, into electric energy. The generators size can be from the one of washing machine to building size, but the bigger and newer it is, the cheaper is for the electricity to be produced. At this level the electricity is at medium voltage, up to 22kV.
b) *Transmission* (transmission lines, transmission stations) – the process of transmitting big amounts of energy at large distances, from power plants to substations located near the demand centers. This is because power plants are located in rural environment, and electricity is needed in the urban environment the most. Transmission level voltage (HV) is considered to be usually from 140kV

up to 765kV. The voltage depends of the amount of electricity that needs to be transferred to the distribution substations.

c) *Distribution* (substations, consumers, active transformers) – delivery of electricity from transmission centers to individual consumers. At this level (MV), the voltage is lower and is usually less than 33kV. The voltage is further lowered at consumer level, which is usually from 100V to 250V, depending on the country.

d) *Retail* (prices, meters) – the prices are calculated by the price per kWh and the number of Kwh spent during a period of one month. The meters installed in households or other facilities are used to measure the amount of electricity spent by the consumer each month.

Figure 27 shows how the electricity is transmitted and distributed from generation point to the different customers, with the different voltages on each level.

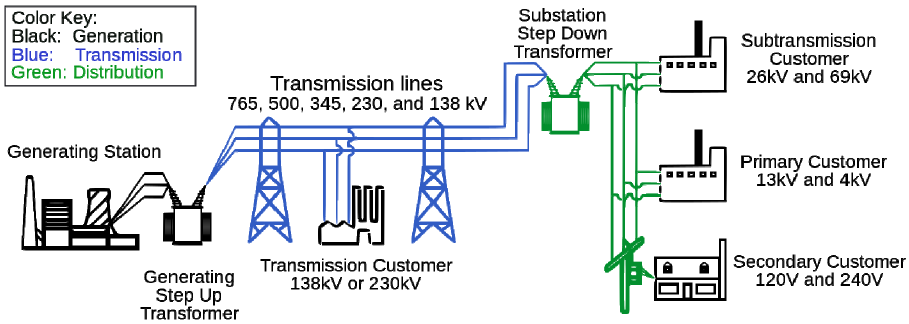

Figure 27 Electricity transmission and distribution scheme

Image source: http://energy.org/cat00040870.html

Reliable operation of the power grid is complex and demanding for two fundamental reasons [25]:

- First, electricity flows at close to the speed of light (186,000 miles per second or 297,600 km/sec) and is not economically storable in large quantities. Therefore electricity must be produced the instant it is used.
- Second, without the use of control devices too expensive for general use, the flow of alternating current (AC) electricity cannot be controlled like a liquid or gas by opening or closing a valve in a pipe, or switched like calls over a long distance telephone network. Electricity flows freely along all available paths from the generators to the loads in accordance with the laws of physics—

dividing among all connected flow paths in the network, in inverse proportion to the impedance (resistance plus reactance) on each path.

Maintaining reliability is a complex enterprise that requires trained and skilled operators, sophisticated computers and communications, and careful planning and design. The North American Electric Reliability Council (NERC) and its ten Regional Reliability Councils have developed system operating and planning standards for ensuring the reliability of a transmission grid that are based on seven key concepts:

- Balance power generations and demand continuously – electricity generation must be constantly fine-tuned in order to me the ever changing demand. The demand is predictable to some extent, with a daily curve that peaks in the afternoon and evening, and higher during weekdays than weekends. If generation fails to match demand, the frequency of an AC power system increases when generation is higher or decreases when demand is higher. Small variations of frequencies are normal and do not influence the system much, but large changes in frequencies lead to equipment damage. In extreme cases, the system implements automatic load shedding, forcing rolling blackouts in order to prevent the whole system to collapse.
- Balance reactive power supply and demand to maintain scheduled voltages – the generator produces two types of electricity: real and reactive, both necessary and balanced. The real power is the one that powers equipment and is measured in watts. Reactive power is the energy supplied to create or be stored in electric or magnetic fields in and around the electrical equipment. This power is measured in volt-ampere reactive – VAr and is important for equipment that relies on magnetic fields for functioning (pumps, AC, motors). If reactive power cannot be supplied when needed and in needed quantity, in extreme cases there can be a voltage collapse. This is why reactive power sources are needed to maintain voltage levels. Low voltage can result in system instability and shutdown, while high voltage can damage lines insulation and cause flashovers.
- Monitor flows over transmission lines and other facilities to ensure that thermal limits are not exceeded – since electricity flows freely across all connected circuits, the power flow changes constantly on distribution and transmission lines. All equipment that carries electricity is heated by its flow, thus raising the need to limit the flow in order to prevent overheating which can lead to damaging equipment. The overhead lines are particularly monitored to prevent overheat-

ing, since the metal can stretch and allow the lines to sag, thus increasing the risk of causing fires or damaging equipment.

- Keep the system in a stable condition – stability problems can develop very quickly – in a matter of seconds. This is why stability limits are imposed in order to prevent system collapse. Voltage stability limits that prevent voltage levels to drop. Power stability limits are set in order not to lose synchronization between generators and loads when there is an unplanned loss of line or shortage.
- Operate the system so that it remains in a reliable condition even if a contingency occurs, such as the loss of a key generator or transmission facility – this means the system must be operated in a preventive more so that when the most important generator or transmission facility fails, the remaining facilities can continue operating.
- Plan, design and maintain the system to operate reliably – short term planning for daily and weekly operations and long term planning for providing generation resources and transmission capacity are both needed to provide system reliability. A utility should also predict future loads in order to arrange sources to meet demand.
- Prepare for emergencies – for rare events, such as severe weather conditions, emergency procedure have to be developed and operators must be trained to recognize and take corrective actions.

One of the big advantages of this vertically integrated system was the ability to exploit economies of scale, which meant that the more electricity produced; the cheaper the price is per unit. On the other hand, this company is responsible to deliver electricity to all consumers in the country, no matter where they are located and whether it is viable for the company to deliver the electricity per unit price. Figure 28 shows the vertically integrated market as it was, and still is in some economies [24]. There is one company providing all electricity related transaction services: from generation to delivery and billing the customer. They use the income to cover costs and the remaining is the profit they gain.

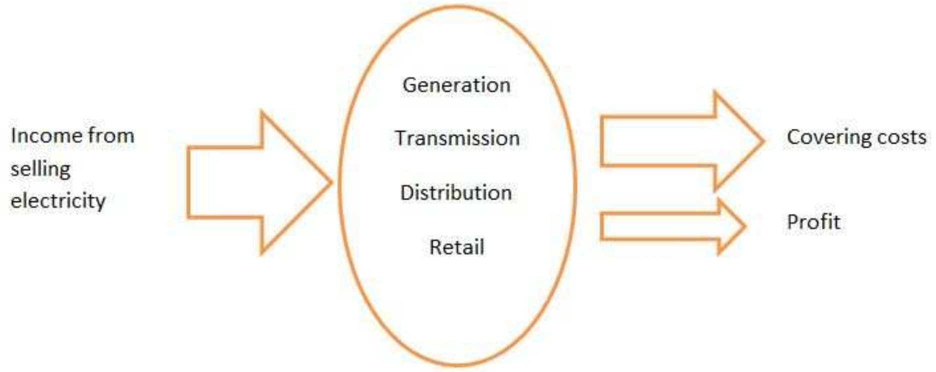

Figure 28 Traditional vertically integrated, regulated company for generating and distributing electricity to its customers.

The traditional way of producing and selling electricity proved to be unfit for the needs of people in the 21st century. It was seen as viable for the electricity industry a century ago, but now it seems like this model is suffocating service innovation.

Today, in developed countries, there is a tendency for electricity market liberalization. This liberalization was being imposed in the hope that it will increase competition among companies, thus lower electricity price. The developed countries were striving towards it, claiming it will be better for society and consumers. The underdeveloped and countries in development, opted for this liberalization in the hope to attract foreign investments by selling public electricity companies. Still there are a lot of developed countries who have not yet adopted this liberalization, because they want to see how it will work out and the advantages from it. Even in countries with liberated electricity markets, it is not fully implemented, if ever.

The original concept of the liberated electricity market was in light with competition; different companies at generation and retail level competing in the market. There is still a vertical integration at the transmission and distribution level, although regulated by the government, since there is no need to double the infrastructure needed to deliver electricity from power plants to end users. Figure 29 explains the envisioned process from generating the energy to getting it to the final customer. The basic idea of a liberated energy market is: competition at wholesale (generation) and retail (service) level, while the transmission and distribution are still vertically integrated, owned by one company and regulated by the government.

With this competition at wholesale and retail levels, the innovation would be encouraged, allowing for better customer service by lowering costs with higher market shares and profits.

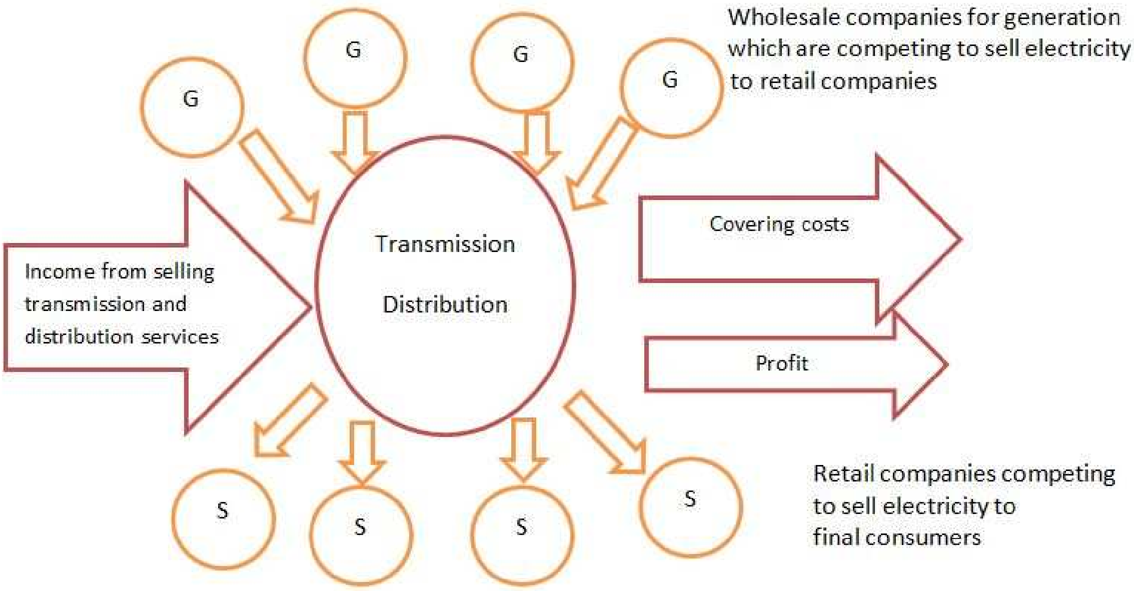

Figure 29 Envisioned de-monopolizing of electricity companies

In reality, the situation was different. In some countries the model developed was very similar to the one in Figure 28, but for the most part the liberalization of the energy market took a slightly different direction. Figure 30 depicts this process for better understanding.

This model depicts the liberalization closer to the one deployed: the monopolized companies from the traditional market become Local Distribution Companies (LDC), with the right to only distribute and sell electricity to end consumers. The generation of electric energy can be done by anyone with the will and finance to do so. The difference between the imagined and deployed model is that in the deployed liberalization there is no competition at retail level, and the monopolized company from the traditional market still makes profit from selling electricity to end users, rather than from giving transmission and distribution services, as was envisioned before.

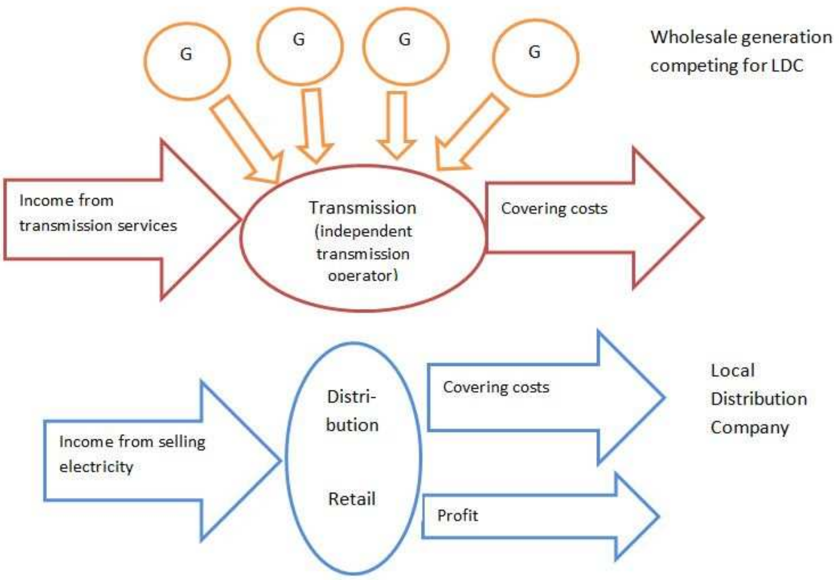

Figure 30 Deregulated market with competing generation companies, transmission is independent and distribution and retail is still monopolized

Figure 31 depicts what really happened with the liberalization of the energy market: characteristic structure of the liberalization as deployed, showing 3 formerly vertically integrated companies for generation and distribution, now big LDCs. Their previous production departments are now independent competing companies, with a new, fourth generation company. The transmission networks, formerly owned by these vertically integrated companies are now connected together in a regional network and regulated by a third independent party – Independent System Operator (ISO).

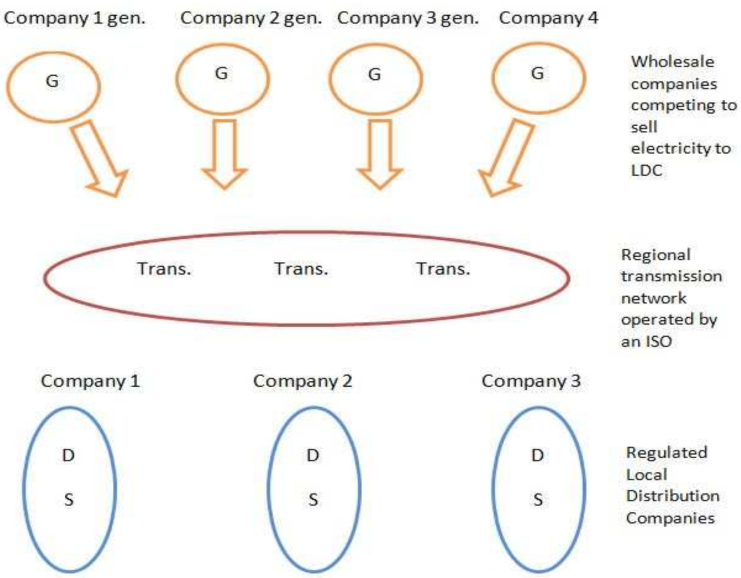

Figure 31 Electricity market after liberalization

In this model, the production companies are competing in the market, and where either a part of a former vertically integrated electricity companies, or completely new formed companies. The transmission system is no longer controlled by the vertically integrated companies, but from and independent ISO. These companies were forced to either sell their transmission lines or rent them to the companies operating them now. The former integrated companies are now only responsible for distribution and selling of electricity to end users, one they buy from the competing production companies.

In the following sections we will see more in depth how the market works at wholesale and retail level, transformation of energy from HV to MV, and then further from MV to LV, as well as the possibilities of buying directly from generators and real time pricing.

2.4.1 Wholesale market operation

The wholesale end of the industry with the liberalization of the electric energy market, works in a competitive model: there are many producers of electricity trying to sell it to the retailers [24]. This is possible by using one or more from the three possible mechanisms: Electricity markets, bilateral trading and/or electricity brokers.

- Electricity markets – this market functions similarly to the stock exchange, there are offers and bidders who want to buy/sell electric energy. The generators state the amount of watts and the price per KWh or the retailers state how many watts they need and the price they are willing to pay per KWh. The market regulates the price as the supply and demand change. Buyers and sellers do not know the identity of the other, or whether the electricity they bought/sold was from/for one party or several. One of the major differences of the electricity market from the stock market is that both parties need to state the availability of electricity they provide/need (for example, I can produce 100MW from 12 to 15 pm tomorrow). Depending on the regulations, electricity can be exchanged only through the electricity market, while other governments allow companies to make transactions through bilateral trading.
- Bilateral trading – this type of trading represents direct exchange of electricity between sellers and buyers. In this case, the identities of both sellers and buyers are known and they communicate directly. One of the main advantages with this exchange, compared to the electricity markets, is that the transaction is flexible,

meaning the period when the electricity is spent as well as the amount is variable. This can be a huge advantage, in the sense that buyers cannot predict demand at all times. These agreements can be for a year upfront, but vary in different periods, according to the agreement and convenience of both sides. Another advantage can be the price, which can be lower than the one traded over the electricity market. Even so, in most regulatory structures, these types of agreements are not allowed, or even if they are the details of the agreement have to be made public and are a subject to veto from the ISO.

- Electricity brokers – these types of brokers make bilateral agreements with wholesale companies for a certain amount of MW in the following year, which they resell to retailers at a higher price. If there is any excess of energy, the broker can always resell this electricity at the electricity market, as well as buy some electricity from the market if the demand from the retailers is higher than the one they have at the moment.

Another difference among markets is whether the retail customers are allowed to state the price they are willing to pay for a certain amount of electricity or they can only choose from the offered prices and amounts. In the first case, every retailer states how many watts they need, at what time and what is the price they are willing to pay for it, and wholesale companies send their offers, the one with the lowest price wins. This is a model called a reverse auction, where there is one buyer with multiple sellers trying to make a transaction. The other model is when the wholesale companies make their offers and the buyer can only choose from those offers, regardless their imagined price per watt, amount or availability. The prices are somewhat regulated by the government, so there will always be some average price to be guided by. In the first case, the wholesale market is more competitive, since the buyer has more power than in the second case. But this is not always the best choice, because if the system becomes too competitive, the small buyers can be left out of the market. Moreover, it makes the market very complicated and even more difficult to be regulated by the ISO.

A different aspect to be considered is whether the generating companies sell energy, capacity or both. Selling the first only means that they sell the energy at a lower price, but they don’t guarantee availability at all times. If there is another retailer willing to pay more for this electricity, they will sell it to them. Selling capacity means availability at all times. They do not actually sell electricity, but if a retail company needs some amount of electricity at certain times, because of an outage or some other fault that their regular provider has, the company is under contract to provide the needed electricity to the retailer. This type of contract is called a reserve contract, and there is a transfer of energy only at certain times. If the wholesale company is selling both, that means that they provide electricity at all times, without shortages at certain times. Depending on the agreement the producer and buyer have, we have different types of contracts: stable, reserve, discontinued or no contract. These options are presented in Table 1.

Table 1 Different types of contracts depending on transaction of capacity and/or energy

Buying capacity? \ Buying energy?	Yes	No
Yes	Stable contract	Reserve contract
No	Discontinued contract	No contract

Electricity transmission on a wholesale level

In order to get the electricity from the generators to the buyers, there is a need for an electricity network that will distribute it to the wanted location. As was mentioned before, this is usually regulated by an independent system operator. Before the deregulation of the market, every company owned their transmission lines. At the middle of the previous century, companies realized it was better to pool their resources and share the transmission lines because they will reduce costs and be able to grow the transmission capacity. This way they used the transmission lines together, and if there was an outage, they always had another line to use until the broken one was fixed. With the market deregulated, the transmission lines are again controlled by one party – the ISO – and offer services to the generators and buyers to make the transfer of electricity needed. This independent system operator does not own the transmission network, rather it operates it. The ownership of this network is of the government or as it was before the liberalization, it is owned by the formerly vertically integrated companies, now LDCs.

Regardless of the ownership of the transmission lines, they have to provide the following services to the market:

- System security – no matter who controls the transmission lines, they have to provide electricity flow and reduce the capacity outages at a minimum. In the deregulated market this becomes a little more complicated since there are more sellers and buyers, which makes the system more unpredictable.
- Electric energy delivery – the transmission system is there for one purpose only: to coordinate and make sure the sold/bought electricity is delivered.
- Covering costs – for lending its services to the sellers/buyers of electricity, the system has to charge them in order to cover the costs of electricity transmission.
- Fair electricity transmission market – the operator has to ensure that the lines are used fairly to transmit electricity to the competing buyers. Usually this is achieved on a first buy, first serve basis, which allows the participants to reserve the lines in a certain period of time.

2.4.2 Retail energy market

In deregulated markets, the electricity retailers are the ones selling electricity to the end users, except for factories and large institutions, which can continue buying electricity at a wholesale level. The way retailer will compete in this market is very similar to the way phone operators compete, with competitive service plans, advertising, improved technology and customer care, all with the lowest possible prices. The end-use customers range from large manufacturing facilities to household users [24].

In markets where retail is fully deregulated, customers can choose to stay with the incumbent facility, or choose one of the new companies competing at the retail level, as opposed to the monopolized markets where the only choice customers had is either buy electricity from the one provider, or go off grid, which can be a very expensive investment, especially for individual households. Usually market liberalization translates into lower prices for end users, but sometimes it can result in higher prices for households and small businesses.

The main responsibilities of retail companies are:

- Buy electricity on a wholesale level in large quantities and distribute it through the transmission networks.
- Transmit smaller amounts of electricity through the distribution network

- Sell electricity to the end-user customers

- Customer management (call center)

Retail companies usually provide fixed prices to their customers, while baring the risk of changing wholesale prices themselves. The electricity prices can be somewhat market regulated or real time pricing. Even if the prices are not regulated, retail companies are motivated to provide lower prices to their customers; otherwise they will be pushed out by other companies with more competitive prices. On the other hand, real time pricing allows users to take full control of what they pay each month: by knowing the prices each hour of the day, they can decide when to use more electricity and when to lower their consumption. With this so called demand response, the peak traffic can be lowered, lowering costs for retail companies, thus lower costs for end users.

Enhancing retail level services

Since the lower the price, the more competitive the retail company is, it leaves them lower profit margins, thus lower profit. In response, they will offer services that will allow them to increase prices and still be acceptable to end users, and this will range from basic “economical” packets for electricity services to first class level of service which are superior to anything offered on the market, thus most expensive. With this difference in service they can achieve the following:

- Difference in product – their electricity can be the same as the one from other retailers, but their services are quite different
- Profit – their first class packets will allow them to generate higher profit margins, sometimes up to 100% higher.

There are a lot of special services, which are potentially very useful, that the retail company can offer, and can provide high values for the customer. Some of them are the following:

- Flexible user service – some of the users may be satisfied with the basic services a retailer will provide, but for some users 24/7 services are crucial for their business, thus the necessity for flexible user packets with different services.

- Selling final products – some retail companies may offer directly some final services, such as heating and cooling.
- Enhanced customer services – competition makes companies more customer friendly, including aspects as communication, bill delivery, complaints processing and payment.
- Backup power – there is a range of possibilities to provide electricity in events of a blackout, ranging from powering only one computer, to the whole facility for up to four weeks.
- Automation and control – this includes “intelligent” control of high voltage appliances, such as meters and AC, which allows for more precise control than what was used in the 80’s.

Real time based pricing

Real time pricing implies giving information to end users about the cost of price at any given time. This information is usually distributed via email, text messages, phone or online. The electricity tariffs usually change every 15 minutes in respect to demand, supply and market conditions. While during off-peak periods, prices tend to be much lower in this scheme compared to flat-based pricing, during peak periods tariffs are significantly higher. Still, this scheme is effective in incentivizing customers to shape their electricity demand according to the prices. This is usually done with demand response schemes, earlier explained in part 2.2.2.

3 Prosumer cooperation

3.1 Game theory

Game theory is a study of strategic decision making. It models conflict and cooperation among intelligent decision makers. One type of games is the cooperative game, where players collaborate to achieve a greater outcome which is higher than their individual gains. The Shapley theory in particular gives a way to divide this total gains to each player of the game, respectively, depending on their contribution to the game.

3.1.1 Cooperative games

Game theory is divided into two branches: cooperative and non-cooperative theory [27]. These branches differ in the level of individuality of players in the game. In non-cooperative theory, the game is detailed and shows all the moves a player can make. In cooperative theory, on the other hand, the focus is on the outcome of the game which is a shared benefit of the players resulting from their cooperation.

Cooperative or coalitional games are the type of games where the players cooperate among each other to get better results, as opposed to non-cooperative games, when everyone plays for their own benefit only. The logic behind this cooperation is that if each player played by themselves, they would get some value when the game is finished. But if they join together, i.e. make a coalition, they can gain more, thus receive greater benefits, or lower costs. This is usually done in politics, when opposing parties join together to gain greater benefits, than the ones they would achieve by themselves. With this in mind, we can assume that the players making the coalition are not always with same interests or contribute the same value to provide the combined benefit. Also, some coalitions cannot be made without some players, while other players are easily replaceable or add no benefit when added in the coalition. Unlike non-cooperative games, which focus mainly on distributing individual payoffs to players, cooperative games focus on what the players as a group can achieve together rather than individually. These benefits then can be redistributed among the players, depending on different traits each player has.

Depending on the different value each player contributes, they expect to receive a proportional benefit from set coalition. There are players that are vital for the coalition itself to exist. These players are called the core of the coalition.

A cooperative game consists of two elements: (a) a set of players and (b) a characteristic function specifying the value created by different subsets of the players in the game. Formally, let $N = \{1, 2, . . . , n\}$ be the (finite) set of players, and let i, where i runs from 1 through n, index the different members of N. The characteristic function is a function, denoted v, which associates with every subset S of N, a number, denoted v(S). The number v(S) is interpreted as the value created when the members of S come together and interact. In sum, a cooperative game is a pair (N, v), where N is a finite set and v is a function mapping subsets of N to numbers.

Or in other words [28]: a coalitional game with transferable utility is a pair (*N*,*v*), where:

- *N* is a finite set of players, indexed by *i* (*i* being each individual player of the *N* set); and
- *v: 2N* → associates with each coalition *S⊆N* a real-valued payoff *v(S)* that the coalition's members can distribute among themselves. We assume that *v(Ø) = 0*. for every coalition *(S)* that can be made from the number of players *i* that belong to the finite set *N*, up to the total number of the set *N* there is a payoff *v* that can after be distributed among the coalition members. Also, we can make the assumptions that the value of the empty set to be 0. A coalitional game can also be defined with a characteristic cost function, where *c: 2N* → and assume that *c(Ø)=0*. This function represents the cost of each player when they form a coalition to perform some task. This is also known as a cost game. Characteristic functions are assumed to be superaditive, a notion explained later in this section.

Among the main questions when using coalitional game theory are:

- Which coalition to form?
- How should the chosen coalition divide its payoffs among the members

A grand coalition is formed if all the players make a set together. But sometimes this is not profitable. Instead it is better for some players to from coalitions with others, making a set of coalitions. Once they are made, they can decide on distributing the total value that the given coalitions get. The counter-coalition of the grand coalition is called an empty coalition, as it contains no players. If a coalition consists of a number of players less than the total number of players, it is called a *proper subset*. If the coalition is made out of one player, it is called a "*singleton coalition*" [29]. The allocation of the benefits is what each player gets after the end of the game. An allocation is *inefficient* if there is at least one person who can do better, while no other person is worse off. That makes sense – if somebody can do better without anyone else being made worse off, then there is an unrealized potential for benefits in the game. Conversely, the allocation is *efficient* in the Pareto efficiency sense if no-one can be made better off without making someone else worse off. Let say that members of coalition *S* get payoff *v*. Suppose that the members of coalition *S* can join another coalition, *S'*, and get and allocation of payoffs *v'*. These members that choose another coalition are called *defectors*. This happens when set members get greater payoffs in coalition *S'* than they would get in the primary coalition, i.e. $v' > v$. In this case, payoff *v'* "dominates" *v* through coalition *S'*.

Types of players in cooperative game theory

Depending on what they contribute, their bargaining power ad value in general, players can be regular, dummy or veto players.

- - Regular players – players that contribute to the coalition and thus expect some payoff as a result, but are not essential for forming a coalition
- Dummy players – players that when added to the coalition don't contribute any value, but are still valuable as a player in the coalition. Thus, as an investment towards good relations, they get some of the payoff when the game is finished
- Veto player – this type of player is essential for a coalition to exist. For example if there are 5 members that can make coalitions, and they need majority to produce some result, as well as one veto player, the coalition made of 3 members has to contain the veto player as well, since this player can veto the proposition even if there is a coalition of 3 regular members making the majority in the game. This may be a player that does not produce any or much value to the coalition, but the setting of the game can allow big power to this player, thus making him essential to the winning coalition and entitled to a share from the payoff.

Supperadditivity, subadditivity and monotonicity

Characteristic functions are often assumed to be supperadditive, subadditive or monotone [30].

- *Supperadditivity* – a game is superadditive if the value of the coalition is equal or bigger than the sum of values the players would get on their own. So, a game $G=(N,v)$ is superadditive if S and T are disjoint coalitions $(S \cap T=\emptyset)$, and $v(S \cup T) \geq v(S)+v(T)$ for all $S,T \subseteq N$ where $v(S \cup T)$ is the value (payoff) of the coalition, $v(S)$ and $v(T)$ are the values of the individual players.

- *Subadditivity* – a game is subadditive when the coalition value is lower than the sum of the separate players' value. This is usually good when the value itself represents cost, so it is good to have lower "value" or cost when making a coalition. So a game $G=(N,v)$ is subadditive if S and T are disjoint coalitions $(S\cap T=\emptyset)$, and $v(S\cup T)\leq v(S)+v(T)$ *for all* $S,T\subseteq N$
- *Monotonicity* - This means that larger coalitions gain more value. A coalition T that is larger than coalition S, produces higher values, $S\subseteq T\rightarrow v(S)\leq v(T)$. This theorem stands when there is superadditivity, which makes a larger coalition have higher payoffs than smaller coalition. From this we can imply that the grand coalition (consisting of all players) has the highest payoff, and thus, value.

The core

The core is a coalitional game theory solution type. Here we talk more about the stability of a coalition, rather than fairness of distribution of payoffs [31]. We see whether the players are willing to form a grand coalition, or would some of them prefer to form smaller coalitions, i.e. defect from the grand coalition. Sometimes smaller coalitions can be more attractive to players, even if they lead to overall lower value than making a big coalition. Let's consider an example.

Voting core example

We have a parliament that is made up of four political parties, A, B, C and D, which have 45, 25, 15 and 15 representatives (100 in total), respectively. They have to vote to pass a 100 mil spending law and how this money will be redistributed to each party for controlling. For the law to be passed there is a requirement of a majority vote, i.e. a minimum of 51 votes. Hence, if there is no majority, the law is not passed, and each party gets zero money to spend [32].

The Shapley value - calculate the Shapley value for each player. First we create society up by adding each player in a different order. The number of combinations is N! = 4! = 24. We then calculate each player contribution; according to the order they were added.

A AB ABC ABCD $v(A)=0$ $v(B)=v(AB)–v(A)=1$ $v(C)=v(ABC)–v(BC)=0$ $v(D)=v(ABCD)–v(ABC)=0$

A AB ABD ABCD $v(A)=0$ $v(B)=v(AB)–v(A)=1$ $v(C)=v(ABCD)–v(ABD)=0$ $v(D)=v(ABD)–v(AV)=0$

A AC ABC ABCD $v(A)=0$ $v(B)=v(ABC)–v(AC)=0$ $v(C)=v(AC)–v(A)=1$ $v(D)=v(ABCD)–v(ABC)=0$

A AC ACD ABCD $v(A)=0$ $v(B)=v(ABCD)–v(ACD)=0$ $v(C)=v(AC)–v(A)=1$ $v(D)=v(ACD)–v(AC)=0$

A AD ABD ABCD $v(A)=0$ $v(B)=v(ABD)–v(AD)=0$ $v(C)=v(ABCD)–v(ABD)=0$ $v(D)=v(AD)–v(A)=1$

A AD ACD ABCD $v(A)=0$ $v(B)=v(ABCD)–v(ACD)=0$ $v(C)=v(ACD)–v(AD)=0$ $v(D)=v(AD)–v(A)=1$

B AB ABC ABCD $v(A)=v(AB)–v(B)=1$ $v(B)=0$ $v(C)=v(ABC)–v(AB)=0$ $v(D)=v(ABCD)–v(ABC)=0$

B AB ABD ABCD $v(A)=v(AB)–\mathrm{v(B)}=1$ $v(B)=0$ $v(C)=v(ABCD)–v(ABD)=0$ $v(D)=v(ABD)–v(AB)=0$

B BC ABC ABCD $v(A)=v(ABC)–v(BC)=1$ $v(B)=0$ $v(C)=v(ABC)–v(AB)=0$ $v(D)=v(ABCD)–v(ABC)=0$

B BC BCD ABCD $v(A)=v(ABC)–v(BC)=0$ $v(B)=0$ $v(C)=v(BC)–v(B)=0$ $v(D)=v(BCD)–v(BC)=1$

B BD ABD ABCD $v(A)=v(ABD)–v(BD)=1$ $v(B)=0$ $v(C)=v(ABCD)–v(ABD)=0$ $v(D)=v(BD)–v(B)=0$

B BD BCD ABCD $v(A)=v(ABCD)–v(BCD)=0$ $v(B)=0$ $v(C)=v(BCD)–v(BD)=1$ $v(D)=v(BD)–v(B)=0$

C AC ABC ABCD $v(A)=v(AC)–v(C)=1$ $v(B)=v(ABC)–v(AC)=0$ $v(C)=0$ $v(D)=v(ABCD)–v(ABC)=0$

C AC ACD ABCD $v(A)=v(AC)–v(C)=1$ $v(B)=v(ABCD)–v(ACD)=0$ $v(C)=0$ $v(D)=v(ACD)–v(AC)=0$

C BC ABC ABCD $v(A)=v(ABC)–v(BC)=1$ $v(B)=v(BC)–v(C)=0$ $v(C)=0$ $v(D)=v(ABCD)–v(ABC)=0$

C BC BCD ABCD $v(A)=v(ABCD)–v(BCD)=0$ $v(B)=v(BC)–v(C)=0$ $v(C)=0$ $v(D)=v(BCD)–v(BC)=1$

C CD ACD ABCD $v(A)=v(ACD)–v(CD)=1$ $v(B)=v(ABCD)–v(ACD)=0$ $v(C)=0$ $v(D)=v(CD)–v(C)=0$

C CD BCD ABCD $v(A)=v(ABCD)–v(BCD)=0$ $v(B)=v(BCD)–v(CD)=1$ $v(C)=0$ $v(D)=v(CD)–v(C)=0$

D AD ABD ABCD $v(A)=v(AD)–v(D)=1$ $v(B)=v(ABD)–v(AD)=0$ $v(C)=v(ABCD)–v(ABD)=0$ $v(D)=0$

D AD ACD ABCD $v(A)=v(AD)–v(D)=1$ $v(B)=v(ABCD)–v(ACD)=0$ $v(C)=v(ACD)–v(AD)=0$ $v(D)=0$

D BD ABD ABCD $v(A)=v(ABD)–v(BD)=1$ $v(B)=v(BD)–v(D)=0$ $v(C)=v(ABCD)–v(ABD)=0$ $v(D)=0$

D BD BCD ABCD $v(A)=v(ABCD)–v(BCD)=0$ $v(B)=v(BD)–v(D)=0$ $v(C)=v(BCD)–v(BD)=1$ $v(D)=0$

D CD ACD ABCD $v(A)=v(ACD)–v(CD)=1$ $v(B)=v(ABCD)–v(ACD)=0$ $v(C)=v(CD)–v(D)=0$ $v(D)=0$

D CD BCD ABCD $v(A)=v(ABCD)–v(BCD)=0$ $v(B)=v(BCD)–v(CD)=1$ $v(C)=v(CD)–v(D)=0$ $v(D)=0$

Looking at the first of the possible ways to create the grand coalition, we can see how much each player contributes to the coalition. Since in this particular game the only way to receive any payoff is to create a coalition with 51 members, each player will either contribute 1 or contribute nothing. When we add player A first, the marginal contribution is 0, since the number of members in party A is 45 which cannot make a coalition by themselves. If we then add party B with their 25 members, we make the majority (they have 70 members in total), so the contribution of B is 1. The contribution of C and D is 0, because the needed majority was made before they were added, so there is no need for their membership in the coalition. After we see the contribution of each player after the different ways of adding them, we can allocate the benefits they would get according to the Shapley value.

$\varphi_A = (12*1 + 12*0)/4! = 12/24 = 0.5$

$\varphi_B = (4*1 + 20*0)/4! = 4/24 = 0.167$

$\varphi_C = (4*1 + 20*0)/4! = 4/24 = 0.167$

$\varphi_D = (4*1 + 20*0)/4! = 4/24 = 0.167$

When we add these values we sum up to 1. Since the benefits to be redistributed are 100 mil, the Shapley value for all the players is:

Shapley values (50, 16.67, 16.67 16.67)

We can see that, even though player B has more votes than players C and D, they all get the same amount of benefits, i.e. sharing the half that is left when the first half is allocated to player A. This arises the question: can a sub coalition gain by defecting the grand coalition? The answer from this question can be easily seen from the calculations we did to determine the Shapley value. While player A, as the majority player, cannot obtain the 51 votes needed for the bill to pass alone, if he makes a coalition with any other 1 member, can obtain the payoffs, thus gain more than 50 mil when the benefits are divided. Let's look at a game like this one, when player A pairs with only one other player at a time.

$v(A)$=45; $v(B$ =25; $v(AB)$=100

A AB $v(A)$=45 $v(B)=v(AB)–v(A)$=55

B AB $v(A)=v(AB)–v(B)$=75 $v(B)$=25

φ_A= (45+75)/2 = 60

φ_B= (55+25)/2 = 40

Shapley value (60, 40) – each player gets higher benefits than in the grand coalition.

$v(A)$=45; $v(C)$=15; $v(AC)$=100

A AC $v(A)$=45 $v(C)=v(AC)–v(A)$=55

C AC $v(A)=v(AC)–v(C)$=85 $v(C)$=15

φ_A= (45+85)/2 = 65

φ_C = (55+15)/2 = 35

Shapley value (65, 35) – each player gets higher benefits again, and A has the highest payout with this coalition.

What will happen if players B,C and D, decide to make a coalition and divide the payoffs among themselves, leaving the player with the most votes, A, out of the coalition? Let's calculate the Shapley value.

B BC BCD $v(B)$=0 $v(C)$=0 $v(D)$=1

B BD BCD $v(B)$=0 $v(D)$=0 $v(C)$=1

C BC BCD $v(C)$=0 $v(B)$=0 $v(D)$=1

C CD BCD $v(C)$=0 $v(D)$=0 $v(B)$=1

D BD BCD $v(D)$=0 $v(B)$=0 $v(C)$=1

D CD BCD $v(D)$=0 $v(C)$=0 $v(B)$=1

$\varphi_B = (1 + 1)/3! = 2/6 = 0.33$

$\varphi_C = (1 + 1)/3! = 2/6 = 0.33$

$\varphi_D = (1 + 1)/3! = 2/6 = 0.33$

Shapley value (33, 33, 33)

The highest value player A gets if they make a coalition with either C or D. but even if he makes a coalition with B, he would get more than when joining the grand coalition. For player B the best outcome is if he makes a coalition with player A. Even if he chooses to make a coalition with both C and D, he would get higher payoffs than when he was part of the grand coalition. Players C and D, being with the same number of voters, also benefit the most if the individually make a coalition with A. They also have the option to make a coalition together with B, and still make higher payoffs than in the grand coalition.

We can see that all the players have a good incentive to defect the grand coalition and form smaller coalitions, of 2 or 3 players, depending if they form a coalition with A in it (2 players suffice) or if they leave A out of the game (three players needed for a majority vote).

So under what condition will the players be willing to form the grand coalition? The answer is that they will do so if and only if the payment division plan is drawn from a set called the core.

Definition of the core

A payoff vector *x* is in the core of a coalitional game *(N,v)* iff [33]

$$\forall S \subseteq N, \sum_{i \in S x_j} \geq v(S)$$

The sum of payoffs to the players in any subcoalition *S* is at least as much as they could earn on their own. In the previous voting game we saw that the opposite was happening, and that the players were getting less in the grand coalition than what they could have gotten if they defected and formed a subcoalition. But if there are no coalitions under which the players could have gotten more payments, then the payoff vector is in the core.

This theory is similar to the Nash equilibrium, because the agents don't have any profitable deviations, i.e. reasons to defect the coalition when the payoff vector is in the core. The difference with Nash equilibrium is that in the core theory, groups of agents can jointly defect the coalition, making it a stronger notion than the Nash equilibrium, in a sense that we don't think only about unilateral deviations.

Core existence and uniqueness

When thinking about a solution we should always think about two things: is the solution always returning something; and is the result one thing, a sharp recommendation, or does is return multiple things? This is when we think of the core solution existence and uniqueness.

Is the core always non-empty, i.e. does it always exist? The answer is no. We can consider the voting game to find the answer to this question. The coalitions that were more profitable for the players in order to get the majority vote count of 51 were the following: {A,B}, {A,C}, {A,D}, {B,C,D}. if the sum of the payoffs of B, C and D is less than the total of 100 mil, if each one of them decides to make a coalition with A, they have a reason to defect and for their own coalition, leaving player A out. On the other hand, if B, C and D decide to make a coalition and get the entire payoff of 100 mil, leaving A with 0, thus incetivising A to make a coaltion with whichever from B, C or D obtained the lowest payoff. So for this game the core is empty.

Is the core always unique? The answer to this question is again no. lets consider that the winning majority is now 80% insdead of 51. The minimal winning coalitions are now {A,B,C} and {A,B,D}. Now the case is that A and B are required in all winning coalitions, meaning that any complete distribution of the 100 mil among A and B now belongs to the core. This is because even if C and D are not being payed, they cannot make a winning coalition without A and B. The stability of the grand coalition is guaranteed as long as the payoffs are divided among A and B only.

Simple game

A game $G=(N,v)$ is called simple if it is monotone, $v(A)\in\{0,1\}$ for each $A\subseteq N$ and $v(N)=1$. This means that if the value of the coalition is either 0 or 1, it is considered a simple game [34]. The voting game we explained before is a simple game, since we either had 100 million to distribute, or 0, depending on whether we get a majority or not.

A player $i\in N$ is said to be a veto player if he belongs to each winning coalition i.e. $v(N/\{i\})=0$. So if a given coalition doesn't involve the veto player, the value of that coalition is 0. In our example, when the majority needed was 51%, the core was empty, since there were no veto players. In the case when the majority required was 80%, the veto players were A and B, thus the payments were to be distributed only among these members.

- Theorem – in a simple game the core is empty iff there is no veto player. If there are veto players, the core consists of all payoff vectors in which the non-veto players get 0.

Convex games

A game is considered to be convex if for all coalitions (S,T) that are strict subsets of the set N, the value of the union of the coalitions is at least as big as the value that the first can achieve by itself, plus the value of what the second can achieve by itself, minus the amount that the coalition in common between these two, can achieve for itself [32].

For all $S,T\subset N$, $v(S\cup T)\geq v(S)+v(T)-v(S\cap T)$.

The convexity is a stronger notion than superadditivity, since the latter assumed that the intersection between the coalitions is empty, while here we are allowing them to have an intersection and considering the value of this intersection in the equation. So here we take in consideration a set where a player can belong to more than one coalition.

- Theorems – Every convex game has a nonempty core. There is always at least one way to divide the payments among the players in order to support the grand coalition in a way that no subset of the agents will be willing to deviate from the grand coalition to make their own coalition and produce higher benefits. Also, in every convex game, the Shapley value is in the core of the game. So in these games, dividing the game's payoffs in a way that is stable (according to the core definition) and dividing the payoffs in a way that is fair (according to the Shapley value), gives us the same result.

The game we are considering next is a convex one.

The airport game example

Lets consider the following game: several neigbouring cities need an airport capacity, with different cities needing to accommodate aircraft of different sizes [35]. They need to decide whether each of the cities is going to build their own airport, or they will come together and build a regional airport and split the costs among themselves. If a new regional airport is built, the cost will depend on the largest aircraft that the runway can accommodate. Otherwise, each city will have to build their own airport. This situation can be modeled as a coalitional game *(N,v)*, where *N* is the set of cities, and *v(S)* is the sum of the costs of building runways for each city in S minus the cost of the largest runway required by any city in *S*. The cities are A, B, C and D that have a need of building ariports with different sizes of runway. Their individual costs are 8, 11, 13, 18 for each city respectively. Table 2 we can see the distribution of cost to each player.

Table 2 Four cities with their marginal contribution to the overall cost

Aircraft	Adding A	Adding B	Adding C	Adding D	Shapley value
Marginal cost	8	3	2	5	

Cost to A	2				2
Cost to B	2	1			3
Cost to C	2	1	1		4
Cost to D	2	1	1	5	9
Total					18

Shapley value (2,3,4,9) - the cost allocation if the cities decide to build the airport together and split the cost. The cost that will be incured in this case, is lower than if they decide to build 4 airports, one for each city. The grand coalition is the best solution for each of the players, so they don't have an incentive to deviate from it and form a subset coalition.

3.1.2 Fairness (Shapley theorem)

What is a fair way to divide the benefits? It depends on the way we define fairness. According to Lloyd Shapley, the division of the benefits should be according to their marginal contribution, or each member should receive a payoff proportional to how much they add value to the coalition. Sometimes this simple division becomes complicated [36]. Let's say that we have a committee where everyone has to be present in order to pass a suggestion. In this case, $v(N)=1$, $v(S)=0$ and $N \neq S$ where N is the total number of players, S is a coalition when a member is missing. Then, $v(N)–v(N \setminus \{i\})=1$ for every i making each member's marginal contribution 1, thus everyone is essential to generating any value. In this situation, we cannot allocate everyone their marginal contribution. There is a need to come up with some weighting system to allocate the benefits among the players. We can just divide the benefits evenly over to each player, so everyone get $1/N$ of the value. But what happens when the players contribute asymmetrical value to the coalition? The Shapley's axioms give some weighting system proposition which help allocate the value properly.

Shapley axioms

- *Symmetry* – Let's take two members of the total number of players, *i* and *j*. If they always contribute the same amount of value to any given coalition they can be a part of, they are considered to be completely interchangeable. This means that for every coalition *S* that has neither *i* nor *j* in it, if we add i to that coalition we get exactly the same value as if we would add j to the same coalition. The symmetry is expressed as $v(S \cup \{i\})=v(S \cup \{j\})$[37].

  Axiom

  For any *v*, if *i* and *j* are interchangeable then $\varphi_i(N,v)=\varphi_j\ (N,v)$, meaning they should get the same allocation of value φ.

- *Dummy players* - a player *i* is a dummy player, if when added to coalition *S*, he adds no value. So for all $S{:}v(S \cup \{i\})=v(S)$ where *v* is the value of the coalition *S* when player *i* is added.

  Axiom

  For any *v*, if *i* is a dummy player, then $\varphi_i(N,v)=0$, meaning if the player contributes no value, they should get no benefit from the coalition.

- *Additivity* - if we can separate a game in two parts, or have two games and we want to sum up their separate value in a combined way, meaning $v=v_1+v_2$, then we should be able to decompose the payments as well.

  Axiom

  For any two games, v_1 and v_2, $\varphi_i(N,v_1+v_2)=\varphi_i(N,v_1)+\varphi_i(N, v_2)$ for each *i*, where the game (N,v_1+v_2) is defined by $(v_1+v_2)(S)=v_1(S)+v_2(S)$ for every coalition *S*, meaning the allocation of benefits from the two games together should be the same as when we sum up the separate benefits of both individual games.

Shapley theorem

Given a coalitional game *(N,v)*, the Shapley value divides payoffs among players [38] according to (Figure 32 Shapley theoremFigure 32):

$$\varphi_i(N,v) = \frac{1}{N} \sum_{S \subseteq N \setminus \{i\}} |S|!(|N| - |S| - 1)! \left[v(S \cup \{i\}) - v(S) \right]$$

Figure 32 Shapley theorem

It is basicly a marginal contribution calculation, thus we calculate the marginal contribution of i when we add it to a coalition *S* that does not have i in it, then weigh this result with the different possible ways we could have come up with the marginal calculation and then dividing it through all the possible ways that it could have been done. This is done for each player *i* of the set.

Theorem

Given a coalitional game *(N,v)*, there is a unique payoff division *x(v)=φ(N,v)* that divides the full payoff of the grand coalition and that satisfies the *Symmetry*, *Dummy player* and *Additivity* axioms; the *Shapley Value*.

This captures the "marginal contributions" of agent *i*, averaging over all the different sequences according to which the *grand coalition* can be built up.

Building up society (grand coalition)

There are many different ways we can build a society up. For example, lets take a 3 person game and start adding the players to build the whole set [34].

1 12 123 person 1 contributes something, then person 2, then person 3

1 13 123

2 12 123

2 23 123

3 13 123

3 23 123

So we have all these possible ways of creating the grand coalition of the 3 players, depending who we add first to start building the society. Depending on the order of adding each player, we have different marginal contributions from each player. So, for any such sequence, we should look at agent i's marginal contribution when added: *[v(S∪{i})-v(S)]*. Next, we weight this quantitiy by the different *(|S|!)* ways the set *S* could have been formed prior i's addition and by the different ways the other players can be added after i has been added$((|N| - |S| - 1)!)$. Then we sum this result over al possible sets S that are there before i is added, and average by dividing by the number of possible orderings of all agents $|N|!$.

The way to see how each player is adding value to the coalition we can see the previous example. We have $N=3$ players and each are being added in a different order. We have $N!=6$ ways to make the grand coalition. We than calculate the value of player whan when he is added to the coalition.

1 12 123 $v(1)$

1 13 123 $v(1)$

2 21 123 $v(12)–v(2)$

2 23 123 $v(123)–v(23)$

3 31 123 $v(13)–v(3)$

3 32 123 $v(123)–v(23)$

Each of these values is weighted by 1/6. Since the result from the first two is the same, the value is weighted by 1/3, as is the 4th and 6th order. The other 2 are weighted by 1/6. This gives us the total value of the Shapley value and tells us what player 1 should be getting from the game.

Two player game

We have 2 partners sharing their profits. Person 1 produces value 1 while person 2 produces value 2. Together they make a value of 4, thus they get a higher value when they work together rather than working separately [34].

$v(\{1\})=1; v(\{2\})=2; v(\{1,2\})=4$

The possible ways to build the society up i.e. create the grand coalition from the players are only two (N!=2): whether we add person 1 or person 2 first to make the coalition. Then we can calculate the payoff that person 1 should be getting from the coalition, depending on when he was added and thus his marginal contribution towards the coalition.

1 12 $v(1)=1$

2 12 $v(12)–v(2)=4–2=2$

They are weighted by 1/2, since there are 2 ways to build society up. With this we can calculate the payoff that person 1 should be getting.

$\varphi_1=1*1/2+2*1/2=0.5+1=1.5$

This means that φ_2 is going to be 2.5 ($\varphi_2=4-\varphi_1=2.5$).

Shapley value (1.5, 2.5)

From the example we can see that the payoffs are not just divided half each, but are distributed more fairly, by giving person 2 a bigger piece of the total benefits, which is the basic notion of the Shapley value.

3.2 The examined scenario

In this section we elaborate our scenario of prosumers, how they will group and cooperate in order to share electricity with other users, and then resell electricity to the grid, and how these benefits will be shared fairly among them.

3.2.1 System model

The system to be modeled is the following: we have a community of a set I *of* $|I|=10$ prosumers and consumers i, since some of the households are consuming and producing electricity (from rooftop solar systems) and some households only consume electricity. Each i, furthermore belongs to a subset $M,N,K\subset C$, where C is the set of consumption of users. There are different sizes of households with different electricity consumption and are divided in these three groups, depending on their average daily electricity needs: 5kWh, 10kWh and 20kWh consumption per household, where $M=\{1,2,3,4\}$, $N=\{5,6,7,8\}$ and $K=\{9,10\}$. We assume that each user belonging to a group has the same consumption as members in the same group, and different from users of different groups. Since the consumption of each player belonging to a group is different from a member of another group $|M_x|\neq|N_y|\neq|K_z|, \forall x,y,z \in C$. In Figure 33 we can see the grouping by consumption levels.

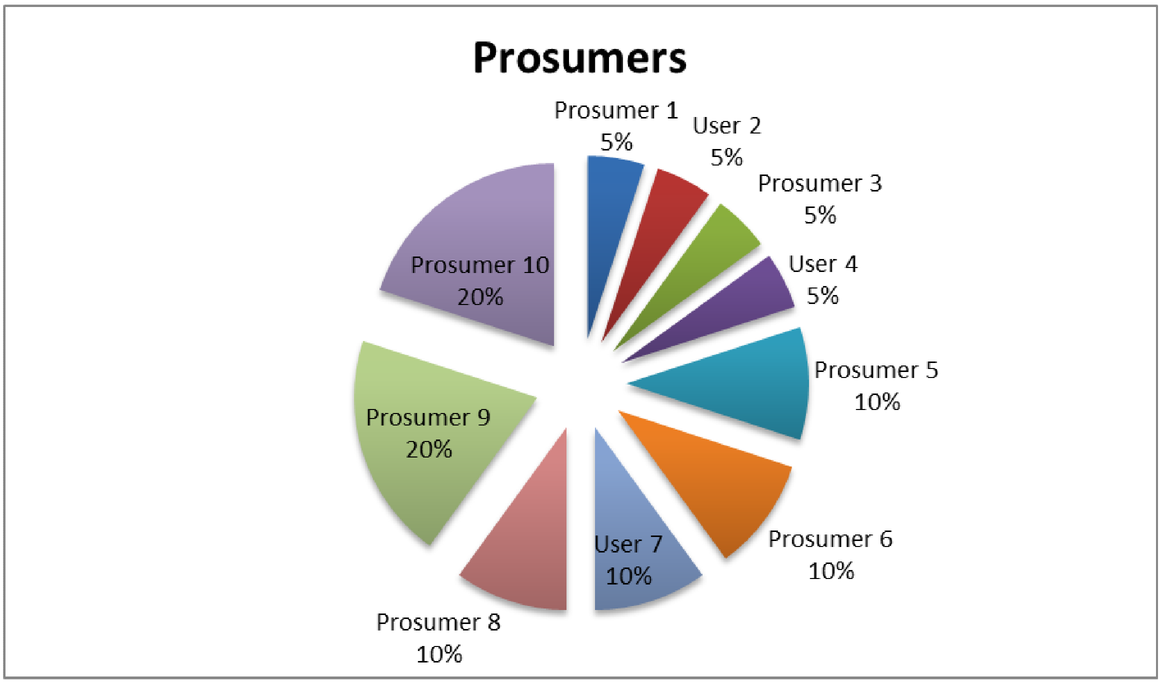

Figure 33 Prosumers and their consumption patterns

Group M – made of 4 users that consume 5kWh per day or 152kWh per month each. Two of them are consumers only, while 2 of them both consume and produce electricity.

Group N – made of 4 users who consume 10kWh per day or average 304kWh per month each. Together they account for 1216kWh electricity spent in the community. In this group, only user 7 does not produce electricity.

Group K – even though this is the smallest group made of 2 users, where both spend 20kWh of electricity daily or 608kWh per month, each, they account for the biggest consumption in the community, consuming as a group 1216kWh of electricity monthly. Both of them are considered prosumers since they also produce energy.

The *amount of energy produced* depends on the *PV system capacity* and the *number of peak sun hours* (PSH) per day. Since the number of effective sun hours is different in different locations, we decided to model this community to be near Thessaloniki. The number of peak sun hours in Thessaloniki throughout the year is different depending on the month of year are presented in Table 3.

Table 3 PSH in Thessaloniki

month	January	February	March	April	May	June	July	August	September	October	November	December
PSH	3.8	4.6	5.4	6.2	8.3	10.2	10.8	9.4	7.5	5.3	3.7	2.7

The users have installed different solar system capacities, each capable of producing different amounts of power. The users that do not produce any capacity are going to use the excess electricity from the other members in the community. The capacity of the panels installed is from 1kW to 4kW. All except one of the prosumers can produce more electricity than they spend on an average monthly. The excess energy they will share it with the non-producing users and users that spend more electricity than they produce.

Installed PV capacities – users have different electricity production capacities based on their needs and possibilities to produce electricity at all. We denote with SC_n the system capacity of prosumer $n \in M, N, K$ where $M,N,K \subset P$, and P denotes the production of each user. There are 5 different types of system capacities SC, where $SC_a=\{2,4,7\}$, $SC_b=\{3\}$, $SC_c=\{1,5,6,8\}$, $SC_d=\{10\}$ and $SC_e=\{9\}$. The production of each group differs from production of users belonging to another group, hence $|SC_a| \neq |SC_b| \neq |SC_c| \neq |SC_d| \neq |SC_e|$, $\forall a,b,c,d,e \in P$.

SC_a (no installed PV capacity) – these are the users that are not prosumers, thus have no installed solar system. The consumers belonging to this group are users 2, 4 and 7.

SC_b (1kW system) – this system is installed by user 3 and produces on average 197.8kWh per month. Depending on the month of year, the lowest electricity produced is 83.7kWh and the highest 334.8kWh.

SC_c (2kW system) – the system installed by user 1, 5, 6 and 8 produces on average 395.6kWh per month. It produces 4747kWh yearly per user. The lowest production is 167.4kWh, while the highest is 669.6kWh.

SC_d (3kW system) – this system is installed by user 10 only, peaking at a production of 1004.4kWh in July, while the lowest production is 251.1kWh in a month. The average monthly production is 593.4kWh.

SC_e (4kW system) – the last installed system in the community is used by user 9. The yearly production is 9494kWh, while the monthly average is 791.7

In Figure 1Figure 34 we can see the monthly average production of each PV system capacity throughout the whole year. We can see that during the summer there is the highest production.

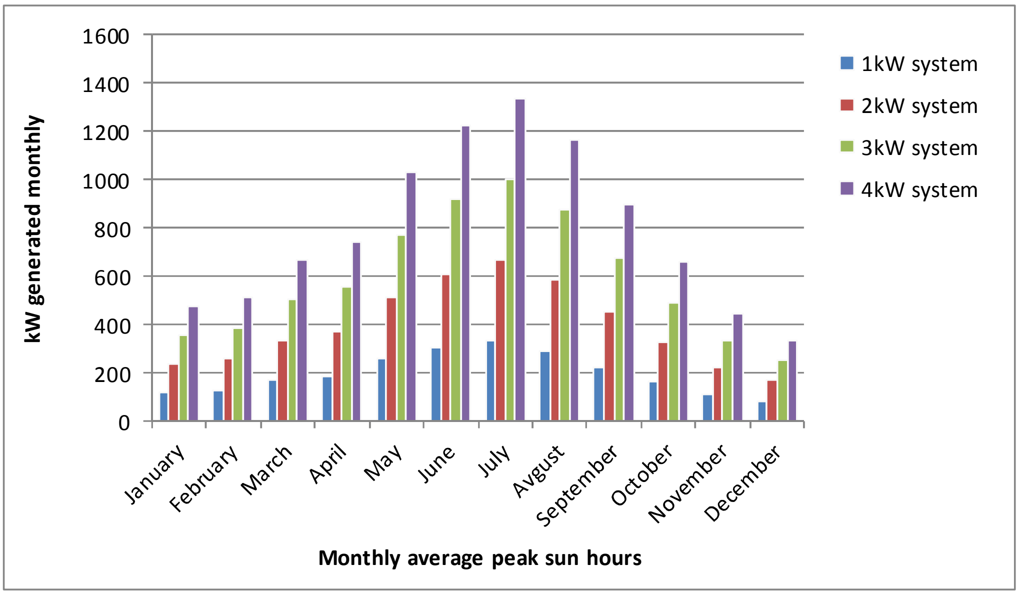

Figure 34 Monthly average production depending on PSH and system PV capacity

In Figure 35 we can see the society we are going to model in this paper. It is presented as a community of houses, where some produce electricity, some don't. The amount of produced and consumed electricity is presented together with their surplus of deficit of electricity. Their total electricity production and consumption is presented as well.

Figure 35 Community of consumers and prosumers

Table 4 denotes all the users, with their production capacity, monthly average consumption and production, as well as the difference between those two in order to have the amount of kWh

Table 4 Users from the community with their consumption, production and difference

Prosumers/Consumers	*Type of PV system installed*	*Monthly average consumption (kWh)*	*Monthly average production (kWh)*	*Difference between production and consumption*
U1	*2kW*	*152*	*395.6*	*+243.6*
U2	*N/A*	*152*	*0*	*-152*
U3	*1kW*	*152*	*197.8*	*+45.8*
U4	*N/A*	*152*	*0*	*-152*
U5	*2kW*	*304*	*395.6*	*+91.6*
U6	*2kW*	*304*	*395.6*	*+91.6*
U7	*N/A*	*304*	*0*	*-304*
U8	*2kW*	*304*	*395.6*	*+91.6*
U9	*4kW*	*608*	*791.7*	*+183.7*
U10	*3kW*	*608*	*593.4*	*-14.6*
Sum		*3040*	*3165*	*+125*

With this scenario, all of the prosumers, except for user 10, will cover their own electricity needs and have excess energy to fulfill the need of the consumers in the community.

3.2.2 Problem statement

The problem we are addressing now is how the users are going to group themselves to achieve the highest benefits. We will model the benefits of the producers with surplus electricity, since they are the ones that should collaborate among each other to satisfy the needs of the consumers in the society.

From the considered consumption and production patterns, we can see that 6 of the users i have a surplus $S=\{1,3,5,6,8,9\}$, and 4 of them have a deficit of electricity $D=\{2,4,7,10\}$. The total deficit is D=622.6 kWh. The total surplus is S=747.7 kWh. We model the prosumers and how their surplus will be distributed to cover the deficit of the consumers. Three of the users $C=\{5,6,8\}$, $C \subset S$, have the same amount of surplus

(91.6 kWh), so they decide to team and act as a single player C in order to gain more benefits, since in calculating the Shapley value, the higher the contribution, the higher the benefits. After the Shapley value is calculated, they distribute the benefits three-way. For this game, we consider the electricity price is the same in the society, which is set at 0.1 euro per kWh. So, the total benefits to be shared by the producing homes will be 62.26 eur. We consider the following players $A, B, C, D \subset S$ where User 1 => A; User 3 => B; User 5,6 and 8 => C; User 9 => D

3.2.3 Proposed algorithm

For the purpose of this model, we assume the following pseudo code as a part of the algorithm we developed to distribute earnings among the players N=4, as seen in Table 5.

Table 5 Model pseudo code

1	*Define prosumers and consumers in the set*	$I = 10,\ i \in I,$
2	*Group users in different sets of consumption habits*	$M,N,K \subset C_n$ $M=\{1,2,3,4\}$, $N=\{5,6,7,8\}$ $K=\{9,10\}$ $\lvert M_x \rvert \neq \lvert N_y \rvert \neq \lvert K_z \rvert,\ \forall x,y,z \in C_n$
3	*Group prosumers in different system capabilities*	SC_n *is system capacity of prosumer* $n \in M,N,K$ *where* $M,N,K \subset P$, *and the production P of each user. There are 5 different types of system capacities SC, where* $SC_a=\{2,4,7\}$, $SC_b=\{3\}$, $SC_c=\{1,5,6,8\}$, $SC_d=\{10\}$ *and* $SC_e=\{9\}$. $\lvert SC_a \rvert \neq \lvert SC_b \rvert \neq \lvert SC_c \rvert \neq \lvert SC_d \rvert \neq \lvert SC_e \rvert,\ \forall a,b,c,d,e \in P$
4	*Calculate electricity produced according to PV system capability and PSH, electricity consumed and difference*	$T_p=\Sigma(SCn*PSH)=3165$, $T_c=\Sigma(M+N+K)=3040$, $T_p-T_c=125kWh$
5	*Define which prosumers have surplus electricity*	$S=\{1,3,5,6,8,9\}$ $D=\{2,4,7,10\}$ *U1=>A, U3=>B, U5,U6,U8=>C, U9=>D* $A,B,C,D \subset S$
6	*Define amount of electricity in surplus and deficit*	*S=747.7 kWh* *D=622.6 kWh*
7	*Define value to be distributed*	*P=0.1 eur v(ABCD)=6226.2*

	among prosumers with surplus electricity	
	Mathematically model prosumers with surplus electricity	
8	*Use Shapley mathematical model to distribute gains among prosumers with surplus*	$\varphi_i(N,v)=\frac{1}{N}\sum_{S\subseteq N\setminus\{i\}}\lvert S\rvert!(\lvert N\rvert-\lvert S\rvert-1)!\left[v(S\cup\{i\})-v(S)\right]$
9	*Find value of each possible coalition*	$G=\{N,v\}$, $N=\{A,B,C,D\}$, $v(ABCD)=62.26;2^N = 2^4=16$
10	*Find possible ways to build the grand coalition*	$N=4; N!=4!=24$
11	*Calculate Shapley value for each player in the grand coalition*	$\varphi_i(N,v)=\frac{1}{N}\sum_{S\subseteq N\setminus\{i\}}\lvert S\rvert!(\lvert N\rvert-\lvert S\rvert-1)!\left[v(S\cup\{i\})-v(S)\right]$
12	*If the grand coalition is stable*	$\forall S\subseteq N,\sum_{i\in Sx_j}\geq v(S)$
13	*End*	
14	*Else*	$v'(S')> v(S), S'\subset S$
15	*Users choose to defect grand coalition*	$d\in S', v'_d>v_d$
16	*Find which coalition can be formed and which player can defect*	$S\{A,C,D\}$
17	*Find veto and dummy players*	*Veto player:* $i\in N, v(N\setminus\{i\})=0$ *Dummy player:* $v(S\cup\{i\})=v(S), \forall v, \varphi_i(N,v)=0$
18	*Find value of each possible coalition*	$G=\{N,v\}$, $N=\{A,,C,D\}$, $v(ACD)=62.26;2^N = 2^3=8$
19	*Find possible ways to build the grand coalition*	$N=3; N!=3!=6$
20	*Calculate Shapley value of each player in new coalition*	$\varphi_i(N,v)=\frac{1}{N}\sum_{S\subseteq N\setminus\{i\}}\lvert S\rvert!(\lvert N\rvert-\lvert S\rvert-1)!\left[v(S\cup\{i\})-v(S)\right]$
21	*Compare Shapley value when players belong to stable set or grand coalition*	
	Consider each player changing their consumption pattern	
22	*When consumption of player A changes, while consumption of other players stays the same*	$B,C,D\in C_n, A'\in C_n', C_n'\subset C_n$

23	*Calculate Shapley value of all players for 4 cases of player A consumption change*	$\varphi_i(N,v)=\frac{1}{N}\sum_{S\subseteq N\setminus\{i\}}\lvert S\rvert!(\lvert N\rvert-\lvert S\rvert-1)![v(S\cup\{i\})-v(S)]$
24	*Make graph of shapley value change of all players*	
25	*When consumption of player C changes, while consumption of other players stays the same*	$A,B,D\in C_n$, $C'\in C_n'$, $C_n'\subset C_n$
26	*Calculate shapley value of all players for 4 cases of player C consumption change*	$\varphi_i(N,v)=\frac{1}{N}\sum_{S\subseteq N\setminus\{i\}}\lvert S\rvert!(\lvert N\rvert-\lvert S\rvert-1)![v(S\cup\{i\})-v(S)]$
27	*Make graph of shapley value change of all players*	
28	*When consumption of player D changes, while consumption of other players stays the same*	$A,B,C\in C_n$, $D'\in C_n'$, $C_n'\subset C_n$
29	*Calculate shapley value of all players for 4 cases of player D consumption change*	$\varphi_i(N,v)=\frac{1}{N}\sum_{S\subseteq N\setminus\{i\}}\lvert S\rvert!(\lvert N\rvert-\lvert S\rvert-1)![v(S\cup\{i\})-v(S)]$
30	*Make graph of shapley value change of all players*	
31	*Calculate difference between total surplus and total deficit of all users in the society*	T_p-T_c=125.3kWh
32	*Calculate leftover electricity of each player according to Shapley value calculated in the grand coalition without changes of electricity consumption by any player*	$v'(N)=v(N)-\varphi_N$, $A,B,C,D\in N$
	Create different scenarios to sell electricity to the power grid, according to price per kWh	

33	*Case 1 – flat selling price*	$\varphi_N=v(N)p$, $p=0.1$
34	*Calculate Shapley value*	$\varphi_i(N,v)=\frac{1}{N}\sum_{S\subseteq N\setminus\{i\}}\lvert S\rvert!(\lvert N\rvert-\lvert S\rvert-1)!\left[v(S\cup\{i\})-v(S)\right]$
35	*Case 2 – linear increase in selling price*	$\varphi_N=\Sigma[v(N)p_\iota]$, $0{,}1>\iota>0{,}2$
36	*Set price for different rounds of selling specific amount of electricity*	
37	*Calculate Shapley value of players for each amount of electricity*	
38	*Sum all values per player to calculate final Shapley value*	
39	*Case 3 – exponential increase in selling price*	$\varphi_N=\Sigma[v(N)p_\iota]$, $0{,}05>\iota>0{,}2$
40	*Set price for different rounds of selling specific amount of electricity*	
41	*Calculate Shapley value*	
42	*Sum all values per player to calculate final Shapley value*	
43	*Case 4 – linear decrease in selling price*	$\varphi_N=\Sigma[v(N)p_\iota]$, $0{,}1>\iota>0{,}2$
44	*Set price for different rounds of selling spe-*	

	cific amount of electricity	
45	*Calculate Shapley value*	
46	*Sum all values per player to calculate final Shapley value*	
47	*Compare Shapley values from all 4 cases and draw conclusions.*	

4 Results

In this chapter we propose a solution to the electricity sharing problem using the Shapley theorem. In the first part we model all players as a part of the grand coalition. Next we examine the stable set and compare the results to the grand coalition values. Then we compute the value using the Shapley theorem for all users when one user changes consumption, while the remaining users' consumption stays constant. We compare results and how the value of each users changes with the electricity surplus change. Next we calculate the electricity that is not consumed by the users in the community, electricity to be sold to the grid via four different pricing schemes. We calculate a fair distribution of payments among players and compare to see which pricing scheme incurs the most benefits to users.

4.1.1 Calculations using the Shapley theorem

We have a 4 player game $G=\{N,v\}$, where $N=\{A,B,C,D\}$ and $v(ABCD)=62.26$ which means that the number of coalitions possible is $2^4=16$ (Table 6). The possible ways to build the grand coalition is 4! = 24 (Table 7).

Table 6 Possible coalitions of players in a game

Coalitions		Value of the characteristic function		
		v	Benefits (euro)	Benefits (kWh)
1	*S = {Ø}*	*v(Ø)*	*0*	*0*
2	*S = {A}*	*v(A)*	*24.36*	*243.6*
3	*S = {B}*	*v(B)*	*4.58*	*45.8*
4	*S = {C}*	*v(C)*	*27.48*	*274.8*
5	*S = {D}*	*v(D)*	*18.37*	*183.7*
6	*S = {A,B}*	*v(AB)*	*28.94*	*289.4*
7	*S = {A,C}*	*v(AC)*	*51.84*	*518.4*
8	*S = {A,D}*	*v(AD)*	*42.73*	*427.3*
9	*S = {B,C}*	*v(BC)*	*32.06*	*320.6*
10	*S = {B,D}*	*v(BD)*	*22.95*	*229.5*
11	*S = {C,D}*	*v(CD)*	*45.85*	*458.5*
12	*S = {A,B,C}*	*v(ABC)*	*56.42*	*564.2*
13	*S = {A,B,D}*	*v(ABD)*	*47.31*	*473.1*
14	*S = {A,C,D}*	*v(ACD)*	*70.21*	*702.1*
15	*S = {B,C,D}*	*v(BCD)*	*50.43*	*504.3*
16	*S = {A,B,C,D}*	*v(ABCD)*	*62.26*	*622.6*

Table 7 Possible ways to build up society

Ways to build up society (grand coalition)	A is added first	B is added first	C is added first	D is added first
N!=4!=24 ways to build the grand coalition	A; AB; ABC; ABCD A; AB; ABD; ABCD A; AC; ACB; ABCD A; AC; ACD;	B; BA; BAC; ABCD B; BA; BAD; ABCD B; BC; BCA; ABCD B; BC; BCD;	C; CA; CAB; ABCD C; CA; CAD; ABCD C; CB; CBA; ABCD C; CB; CBD;	D; DA; DAB; ABCD D; DA; DAC; ABCD D; DB; DBA; ABCD D; DB; DBC;

	ABCD	ABCD	ABCD	ABCD
	A; AD; ADB; ABCD	B; BD BDA; ABCD	C; CD; CDA; ABCD	D; DC; DCA; ABCD
	A; AD; ADC; ABCD	B; BD; BDC ABCD	C; CD; CDB; ABCD	D; DC; DCB; ABCD

After we have calculated all the possible coalition values and the possible ways to create the grand coalition, we can calculate the Shapley value for each player.

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*16.41 + 6*11.83)/24 = (389.76 + 32.82 + 70.98)/24 = 493.56/24 = *20.565*

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*0)/24 = 82.44/24 = *3.435*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-v(ABD))/24] = (6*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*19.53 + 2*27.48 + 6*14.95)/24 = (439.68 + 39.06 + 89.7)/24 = 568.44/24 = *23.685*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*10.42 + 2*18.37 + 6*5.84)/24 = (293.92 + 20.84 + 35.04)/24 = 349.8/24 = *14.575*

Shapley value (20.565, 3.435, 23.685, 14.575)

Check if the coalition is stable

The distribution of benefits we have with the Shapley value is fair, but not stable. The sum of electricity available is more than the electricity needed in the society. As player

B has a small amount to contribute to the coalition, and as we can see that the value of coalition *S{A,C,D}* is more than the electricity needed, these 3 players can decide to form a coalition of their own and share the profit among themselves.

In this case the *Shapley value* would be (21.71, 24.83, 15.72).

Here are the calculations: First we have to find the possible ways to build up society, that in this case with N=3 is 3! = 6 (Table 8), and the possible coalitions, which are $2^N = 2^3 = 8$ (Table 9).

Table 8 Possible coalitions of players in the stable game

Coalition		Value of the characteristic function		
		v	Benefits (euro)	Benefits (kWh)
1	*S = {Ø}*	*v(Ø)*	*0*	*0*
2	*S = {A}*	*v(A)*	*24.36*	*243.6*
3	*S = {C}*	*v(C)*	*27.48*	*274.8*
4	*S = {D}*	*v(D)*	*18.37*	*183.7*
5	*S = {A,C}*	*v(AC)*	*51.84*	*518.4*
6	*S = {A,D}*	*v(AD)*	*42.73*	*427.3*
7	*S = {C,D}*	*v(CD)*	*45.85*	*458.5*
8	*S = {A,C,D}*	*v(ACD)*	*62.26*	*622.6*

Table 9 Possible ways to build up society of stable coalition

Ways to build up society (grand coalition)	A is added first	C is added first	D is added first
N!=3!=6 ways to build stable coalition	A; AC; ACD A; AD; ADC	C; CA; CAD C; CD; CDA	D; DA; DAC D; DC; DCA

After we have the values of each possible coalition and the possible ways to make the stable coalition, we can calculate the Shapley value for each player.

φ_A= 1/3! Σ [(2*v(A) + v(ACD)-v(C) + v(ACD)-v(D) + 2*v(ACD)-v(CD))/6] = (2*24.36 + 2*24.36 + 2*16.41)/6 = (97.44 + 32.82)/6 = 130.26/6 = *21.71*

φ_C= 1/3! Σ [(2*v(C) + v(ACD)-v(A) + v(ACD)-v(D) + 2*v(ACD)-v(AD))/6] = (2*27.48 + 2*27.48 + 2*19.53/6 = (109.92 + 39.06)/6 = 148.98/6 = *24.83*

φ_D= 1/3! Σ [(2*v(D) + v(ACD)-v(A) + v(ACD)-v(C) + 2*v(ACD)-v(AC))/6] = (2*18.37 + 2*18.37 + 2*10.42)/6 = (73.48 + 20.84)/6 = 94.32/6 = *15.72*

The benefits of players A, C and D are higher in this coalition without B than in the grand coalition. This means that the grand coalition does not belong in the core and it is considered unstable. This is also why it is better for users 5, 6 and 8 to act as one player (player C), so they have to be in the set in order for the game to be viable. We can also conclude that these players are considered *veto players*, since any profitable coalition has to contain them in the set. Player B is considered a *dummy player* when added last to the grand coalition since they contribute no value to it.

4.1.2 Calculation using the Shapley theorem when players change consumption behavior

We consider the change of players' Shapley value when one of the players changes their consumption amount, thus changing their surplus of electricity and contribution to the coalition. In each case, the consumption of other players remains constant.

Player A changes consumption

Let's consider what happens to the Shapley value of all players, when one player changes their consumption, while others' consumption and surplus stays the same. In Table 10 we can see how player's A surplus changes when they change their consumption levels.

Table 10 Change in A's consumption leads to different possible benefits

Monthly average consumption of player A	Monthly average production of player A	Surplus of player A's electricity and possible benefits	
		v(A) kWh	v(A) Euro

182.5	395.6	213.1	21.31
212.9	395.6	182.7	18.27
243.3	395.6	152.3	15.23
273.7	395.6	121.9	12.19

Now we calculate the Shapley value for each player, which we expect to be different depending on the 4 values of player A.

a) *v(A) = 21.31 Shapley value (18.532, 3.435, 24.701, 15.592)*

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*21.31 + 2*21.31 + 2*21.31 + 2*21.31 + 2*21.31 + 2*21.31 + 2*16.41 + 6*11.83)/24 = (340.96 + 32.82 + 70.98)/24 = 444.76/24 = *18.532*

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*0)/24 = 82.44/24 = *3.435*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-v(ABD))/24] = (6*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*22.58 + 2*27.48 + 6*18)/24 = (439.68 + 45.16 + 108)/24 = 592.84/24 = *24.701*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*13.47 + 2*18.37 + 6*8.89)/24 = (293.92 + 26.94 + 53.34)/24 = 374.2/24 = *15.592*

b) *v(A) = 18.27 Shapley value (16.505, 3.435, 25.715, 16.605)*

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*18.27 + 2*18.27 + 2*18.28 + 2*18.27 + 2*18.27 + 2*18.27 + 2*16.41 + 6*11.83)/24 = (292.32 + 32.82 + 70.98)/24 = 396.12/24 = *16.505*

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*0)/24 = 82.44/24 = *3.435*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-v(ABD))/24] = (6*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*25.62 + 2*27.48 + 6*21.04)/24 = (439.68 + 51.24 + 126.24)/24 = 617.16/24 = *25.715*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*16.51 + 2*18.37 + 6*11.93)/24 = (293.92 + 33.02 + 71.58)/24 = 398.52/24 = *16.605*

c) *v(A) = 15.23 Shapley value (14.38, 3.73, 26.63, 17.52)*

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*15.23 + 2*15.23 + 2*15.23 + 2*15.23 + 2*15.23 + 2*15.23 + 2*15.23 + 6*11.83)/24 = (274.14 + 70.98)/24 = 345.12/24 = ***14.38***

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*1.18)/24 = (82.44 + 7.08)/24 = 89.52/24 = *3.73*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-v(ABD))/24] = (6*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 6*24.08)/24 = (494.64 + 144.48)/24 = 639.12/24 = *26.63*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 6*14.97)/24 = (330.66 + 89.82)/24 = 420.48/24 = *17.52*

d) *v(A) = 12.19 Shapley value (12.1, 4.49, 27.39, 18.28)*

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*12.19 + 2*12.19 + 2*12.19 + 2*12.19 + 2*12.19 + 2*12.19 + 2*12.19 + 6*11.83)/24 = (219.42 + 70.98)/24 = 290.4/24 = *12.1*

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*4.22)/24 = (82.44 + 25.32)/24 = 107.76/24 = *4.49*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-v(ABD))/24] = (6*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 2*27.48 + 6*27.12)/24 = (494.64 + 162.72)/24 = 657.36/24 = *27.39*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 6*18.01)/24 = (330.66 + 108.06)/24 = 438.72/24 = *18.28*

After calculating the Shapley values of all players in all four scenarios, we can plot them on a graph to see their behavior according to player's A consumption change. In Figure 36 we can see opposite movements of the value, as A's value decreases, the other players' values increase, with B's value increasing slightly. This happens because B does not contribute much to the coalition, and in most of the cases can be left out of the winning coalition.

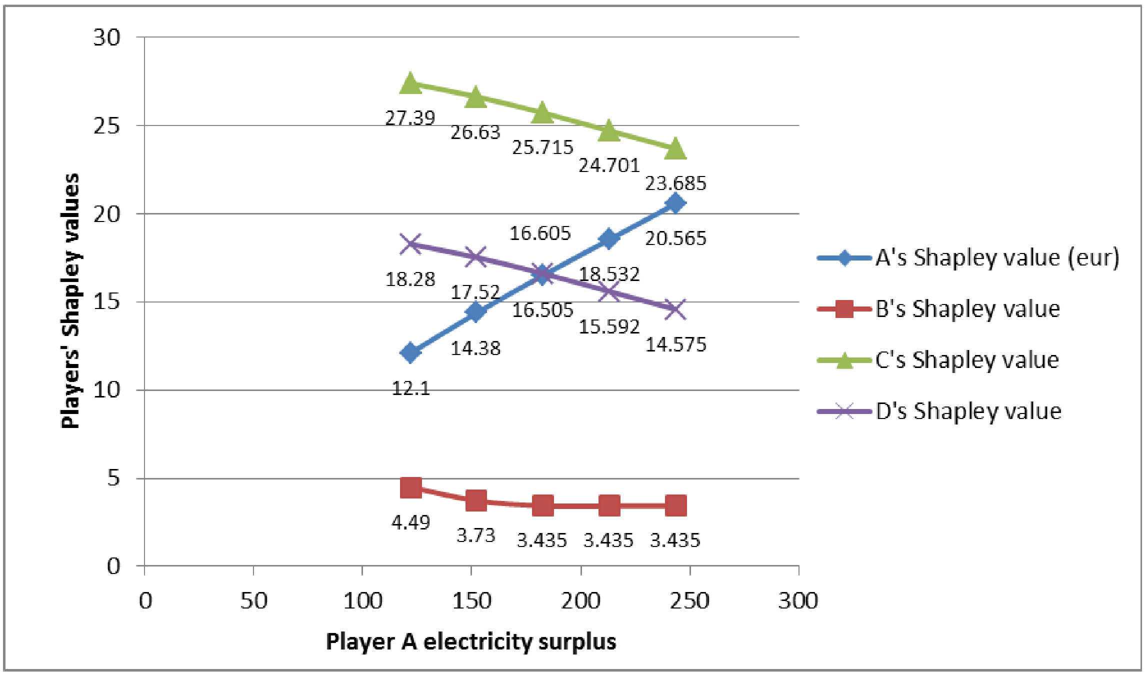

Figure 36 Shapley value of all players in the game and how it changes when player's A electricity surplus changes

The Shapley value increases for players B, C and D, as the contribution of A decreases. After the point of A contributing 152.3 kWh to the society, the set becomes *stable*, and no one is considered to be a *veto player*, as well as no one can do better if they deflect the grand coalition. When A's contribution, as well as benefits are higher than this, he, together with players C and D can deflect the grand coalition and obtain higher benefits than in the grand coalition.

Player C changes consumption

What happens when player's C (a player that is a group of 3 users from the society) electricity needs increase, thus the value contributed decreases? We can see the possible changes in Table 11.

Table 11 Change in C's consumption leads to different possible benefits

Monthly average consumption of player C	Monthly average production of player C	Surplus of player C's electricity and possible benefits	
		v(C) kWh	v(C) Euro

939.87	395.6*3 = 1186.8	246.9	24.69
958.12	1186.8	228.6	22.86
976.37	1186.8	210.4	21.04
1003.75	1186.8	183	18.3

Since the production of C is made of 3 players, we sum the consumption of all 3 players and then subtract their total consumption in order to calculate their surplus. This surplus is then multiplied by 0.1 eur, which is the price per kWh. Now we can calculate the new Shapley values when these changes are taken into account.

a) v(C) = 24.69 Shapley value (21.495, 3.435, 21.825, 15.505)

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*19.19 + 6*14.62)/24 = (389.76 + 38.38 + 87.72)/24 = 515.86/24 = *21.495*

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*0)/24 = 82.44/24 = *3.435*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-v(ABD))/24] = (6*24.69 + 2*24.69 + 2*24.69 + 2*24.69 + 2*24.69+ 2*19.53 + 2*24.69 + 6*14.95)/24 = (395.04 + 39.06 + 89.7)/24 = 523.8/24 = *21.825*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*13.21 + 2*18.37 + 6*8.63)/24 = (293.92 + 26.42 + 51.78)/24 = 372.12/24 = *15.505*

b) *v(C) = 22.86 Shapley value (22.105, 3.435, 20.605, 16.115)*

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*21.03 + 6*16.45)/24 = (389.76 + 42.06 + 98.7)/24 = 530.52/24 = *22.105*

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*0)/24 = 82.44/24 = *3.435*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-v(ABD))/24] = (6*22.86 + 2*22.86 + 2*22.86 + 2*22.86 + 2*22.86 + 2*19.53 + 2*22.86 + 6*14.95)/24 = (365.76 + 39.06 + 89.7)/24 = 494.52/24 = *20.605*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*15.04 + 2*18.37 + 6*10.46)/24 = (293.92 + 30.08 + 62.76)/24 = 386.76/24 = *16.115*

c) v(C) = 21.04 Shapley value (22.712, 3.435, 19.392, 16.721)

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*22.85 + 6*18.27)/24 = (389.76 + 45.7 + 109.62)/24 = 545.08/24 = *22.712*

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*0)/24 = 82.44/24 = *3.435*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-

v(ABD))/24] = (6*21.04 + 2*21.04 + 2*21.04 + 2*21.04 + 2*21.04 + 2*19.53 + 2*21.04 + 6*14.95)/24 = (336.64 + 39.06 + 89.7)/24 = 465.4/24 = *19.392*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*17.76 + 2*18.37 + 6*12.28)/24 = (293.92 + 33.72 + 73.68)/24 = 401.32/24 = *16.721*

d) v(C) = 18.3 Shapley value (23.522, 3.742, 17.463, 17.533)

φ_A= 1/4! Σ [(6*v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(BC) + 2*v(ABCD)-v(BD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(BCD))/24] = (6*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 2*24.36 + 6*21.01)/24 = (438.48 + 126.06)/24 = 564.54/24 = *23.522*

φ_B= 1/4! Σ [(6*v(B) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(CD) + 6*v(ABCD)-v(ACD))/24] = (6*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 2*4.58 + 6*1.23)/24 = (82.44 + 7.38)/24 = 89.82/24 = *3.742*

φ_C= 1/4! Σ [(6*v(C) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(D) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AD) + 2*v(ABCD)-v(BD) + 6*v(ABCD)-v(ABD))/24] = (6*18.3 + 2*18.3 + 2*18.3 + 2*18.3 + 2*18.3 + 2*18.3 + 2*18.3 + 6*14.95)/24 = (329.4 + 89.7)/24 = 419.1/24 = *17.463*

φ_D= 1/4! Σ [(6*v(D) + 2*v(ABCD)-v(A) + 2*v(ABCD)-v(B) + 2*v(ABCD)-v(C) + 2*v(ABCD)-v(AB) + 2*v(ABCD)-v(AC) + 2*v(ABCD)-v(BC) + 6*v(ABCD)-v(ABC))/24] = (6*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 2*18.37 + 6*15.02)/24 = (330.66 + 90.12)/24 = 420.78/24 = *17.533*

Now that we have calculated the new Shapley values of all players according to the change in consumption of player C, we can see how these changes relate to players benefits (Figure 37). As C’s electricity consumption increases, the Shapley value decreases linearly. The other players’ value increases, although we can see that for the bigger players the increasing is more than the small players (as noted in player B, where they increase their benefit only in the last case.)

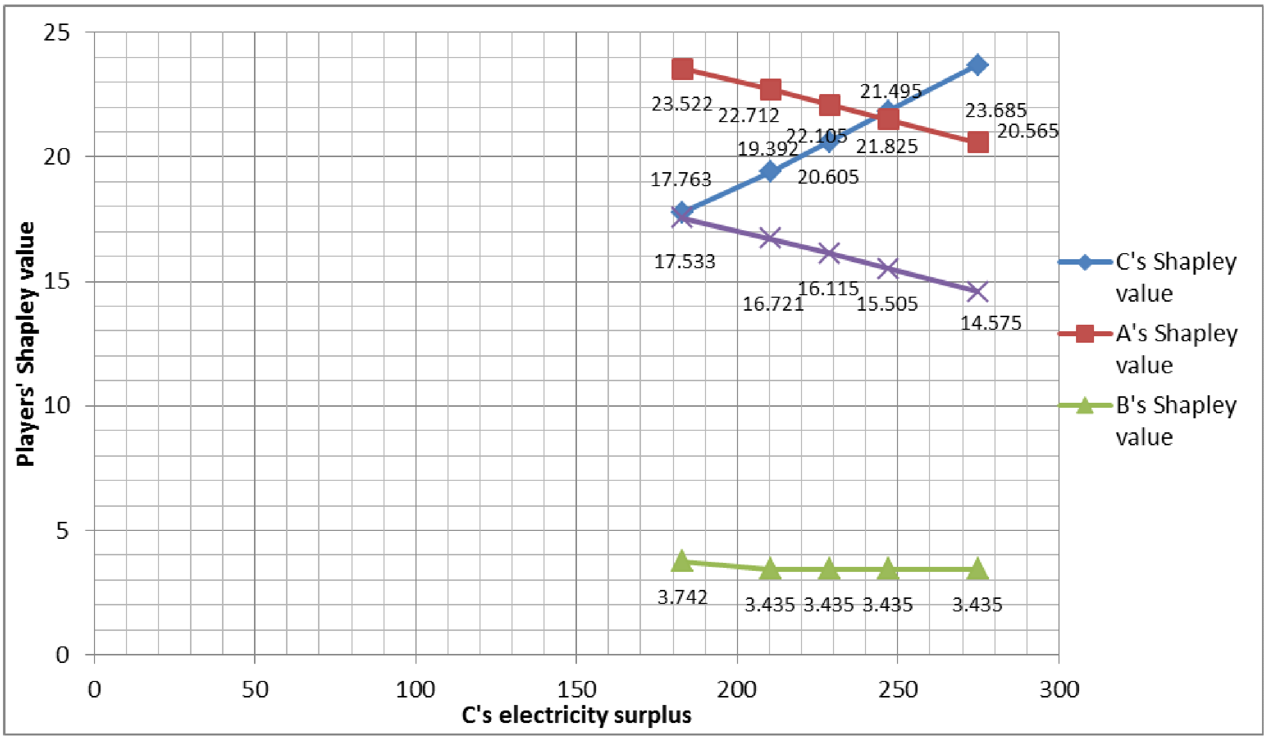

Figure 37 Shapley value of all players in the game and how it changes when player's C electricity surplus changes

Player D changes consumption

First we denote how the change in consumption influences player's D surplus of electricity (Table 12). Then we calculate the Shapley value of each player in the different scenarios. At the end we plot the different Shapley values on a graph to see the change in value, whether it increases or decreases according to increase/decrease of consumption level. The calculations are the same as in the two examples before.

Table 12 Change in D's consumption leads to different possible benefits

Monthly average consumption of player D	Monthly average production of player D	Surplus of player D's electricity and possible benefits	
		v(D) kWh	v(D) Euro
638.7	791.7	153	15.3
669.2	791.7	122.5	12.25

699.6	791.7	92.1	9.21
730	791.7	61.7	6.17

Next we can calculate the Shapley value of each player. (since the calculations are the same only with different values, they are not presented for this part).

a) v(D) = 15.3 Shapley value (21.589, 3.435, 24.708, 12.528)

b) v(D) = 12.25 Shapley value (22.605, 3.435, 25.725, 10.495)

c) v(D) = 9.21 Shapley value (23.517, 3.738, 26.637, 8.368)

d) v(D) = 6.17 Shapley value (24.277, 4.498, 27.397, 6.088)

Next we can plot the changed values to see the behavior when D's surplus changes. From Figure 38 we can conclude that again as one players consumption increases, the values of other players decreases.

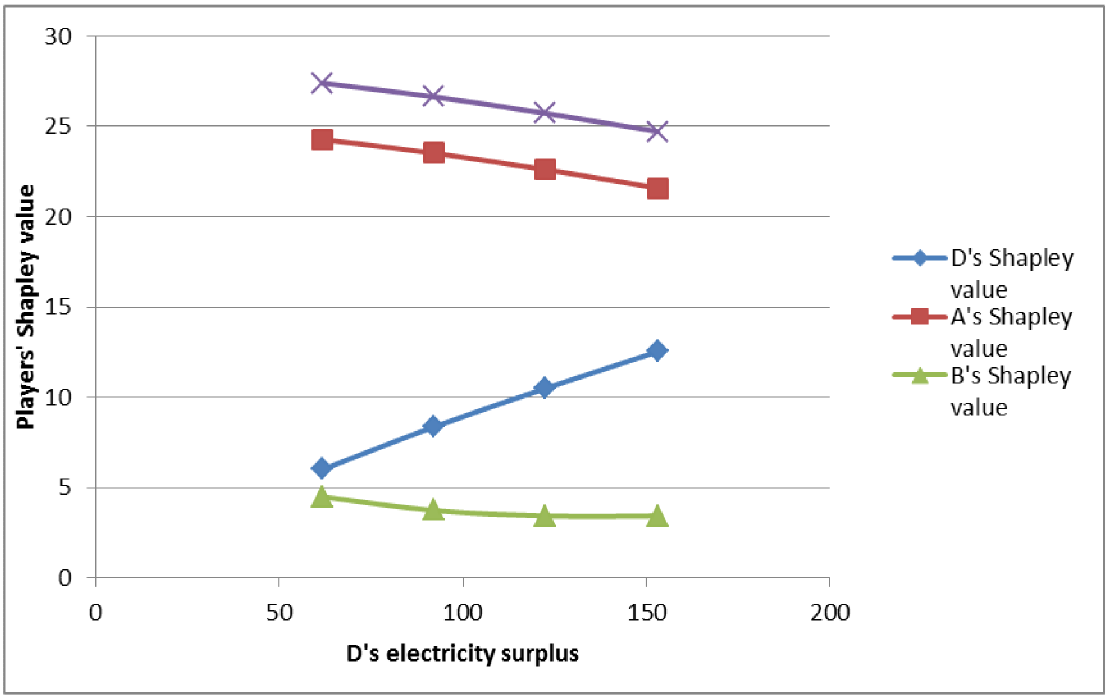

Figure 38 Shapley value of all players in the game and how it changes when player's D electricity surplus changes

4.1.3 Calculations using Shapley theorem when players sell excess electricity to the grid using different pricing schemes

The amount of electricity that will be covering the needs of the consumers in the society is more than what is produced. This difference is the amount to be sold to the power grid which is T_p-T_c=$125.3kWh$. Instead of being wasted, the electricity is sold to the grid. We assume that different utilities offer different pricing schemes: flat price, linear price increase, exponential price increase and linear price decrease. First we calculate which amount from this remaining electricity belongs to which player. We achieve this by subtracting the total surplus of each player's electricity *v(N)* with the electricity they sold to their neighboring consumers φ_N; $v'(N)=v(N)-\varphi_N$, $A,B,C,D \in N$.

Shapley value (20.565, 3.435, 23.685, 14.575)

$v'(A)=v(A)-\varphi_A=243.6 – 205.65 = 37.95kWh$

$v'(B)=v(B)-\varphi_B=45.8 – 34.35 = 11.45kWh$

$v'(C)=v(C)-\varphi_C=274.8 – 236.85 = 37.95kWh$

$v'(D)=v(D)-\varphi_D=183.7 – 145.75 = 37.95kWh$

After we have calculated what each player will be contributing to the game, we consider four cases of selling profiles, each with different pricing schemes: flat selling price, linear increase in price, exponential increase in price and linear decrease in price. In each case, except the first one, we divide the total amount in groups with different price, which increases/decreases after a certain amount of electricity, depending on the pricing model. For each group of pricing scheme we calculate the Shapley value for each player, thus dividing the profits fairly among the players. We do this because if let say two players sell their electricity for price *x* and two players sell their electricity price for price *y*, and $x>y$, the first two players would unfairly benefit from the transaction. Hence the need for the fairness of the Shapley theorem to distribute benefits according to contribution of each player.

Case 1 – Flat selling price

In this case, we have the same price as when selling in the grid (p=0.1 eur per kWh). The amount of electricity sold to the grid does not affect the price, so it doesn't matter how much they will sell, the price will remain constant. This makes the prosumer equally motivated to sell to the grid or to the community. And since the price is constant, each player receives benefits of $\varphi_N = v*p$ where v is the value of a player and p is the price.

Shapley value (3.795, 1.145, 3.795, 3.795)

The players' Shapley values are denoted in Table 13.

Table 13 Shapley values of all players in euros

Shapley value in euros	*v(A)*	*v(B)*	*v(C)*	*v(D)*
Price (0.1)	*3.795*	*1.145*	*3.795*	*3.795*

Case 2 – Linear increase of price

This means the more they sell the price per kWh increases linearly. So if they sell 50kWh they will get 0.1 eur per kWh, for the next 50 they will get 0.15 eur and so on. This is a more motivating scheme for the prosumers since they receive more as they sell more. In this case we calculate the Shapley value for each player in a given pricing scheme, and then multiply the value of the given price in set group. When we calculate the value in euros for each player in all groups, we sum to get the total value of each player. This is done so there will be a fair distribution of benefits to all players. The calculations are done in a more simple way since they are the same as in the previous cases, so there is no need for a step by step explanation.

a) *First 50kWh with a price of 0.1 eur per kWh*

Shapley value (3.795, 1.145, 3.795, 3.795)

$\varphi_A = \varphi_C = \varphi_D$=(37.95*6 + 12.05*4 + 37.95*2 + 0.6*6 + 0*6)/24 = (303.6 + 48.2 + 3.6)/24 =14.75 kWh=*1.48eur*

φ_B=(11.45*6 + 11.45*6 + 0*12)/24 = 5.725kWh = *0.572 eur*

b) *Second 50 kWh with price of 0.15 eur per kWh*

It is important to note that the values of each player decreases in each round of calculations. So now we take into account the values calculated by subtracting the total value of each player with the Shapley value calculated in the previous round of 50kWh sold

v(A)=v(C)=v(D)=37.95-14.75=23.2kWh

v(B)=11.45-5.725=5.72kWh

Shapley value (2.31,0.564,2.31,2.31)

$\varphi_A = \varphi_C = \varphi_D$=(23.2*6 + 23.2*6 + 3.6*2 + 21.08*4 + 0*6)/24 = (278.4 + 7.2 + 84.52)/24 = 15.41kWh = *2.31 eur*

φ_B=(5.72*6 + 5.72*6 + 3.6*6 + 0*6)/24 = (68.64 + 21.6)/24 = 3.76kWh = *0.564 eur*

c) Last 25.3 kWh with a price 0.2 eur per kWh

v(A)=v(C)=v(D)=23.2-15.41=7.79kWh

v(B)=5.72-3.76=1.96kWh

Shapley value (1.558,0.392,1.558,1.558)

$\varphi_A = \varphi_C = \varphi_D$=7.79kWh = *1.558 eur*

φ_B=*1*.96kWh = *0.392 eur*

We can see the final benefits that users receive after selling all their electricity in Table 14.

Table 14 Shapley value of players in a linear increase pricing scheme

Shapley value in euros	*v(A)*	*v(B)*	*v(C)*	*v(D)*
Price (0.1)	*1.48*	*0.572*	*1.48*	*1.48*
Price (0.15)	*2.31*	*0.564*	*2.31*	*2.31*
Price (0.2)	*1.558*	*0.392*	*1.558*	*1.558*
Total	*5.348*	*1.528*	*5.348*	*5.348*

Case 3 – Exponential price increase

Another pricing scheme denotes that the more they sell, the price will grow exponentially. If they sell 20 kWh they will be paid 0.05 eur per kWh, for the next 40 they will get 0.1 eur, for the next 80 they will get 0.2 eur. This motivates prosumers to produce more and use less electricity so they can earn more by obtaining higher prices per kWh when they sell larger amounts to the grid. It also motivates them to sell more to the grid and less to the community. We make the same calculations as in the previous case, with the difference of the amount of kWh in each group. Since this scheme allows prices to grow faster than in the previous case, the amount also increases rather than be equal. Another difference is the starting point of the price for the first pricing scheme, which is lower, thus giving benefits when there is a larger amount of electricity sold.

a) First 20kWh with a price of 0.05 eur per kWh

$\varphi_A = \varphi_C = \varphi_D$=(20*6 + 8.55*2 + 0*16)/24 = (120 + 17.1)/24 = 5.71kWh = *0.285eur*

φ_B=(11.45*6 + 0*18)/24 = 68.7/24 = 2.87kWh = *0.143 eur*

b) Second 40kWh with a price of 0.1 eur per kWh

v(A)=v(C)=v(D)=37.95-5.71=32.24kWh

v(B)=11.45-2.87=8.58 kWh

$\varphi_A = \varphi_C = \varphi_D$=(32.24*6 + 7.76*4 + 31.42*2 + 0*12)/24 = (193.44 + 31.04 + 62.84)/24 = 11.97 kWh = *1.197 eur*

φ_B=(8.58*6 + 7.76*6 + 0*12)/24 = (51.48 + 46.56)/24 = 4.09 kWh = *0.409 eur*

c) Last 65.3kWh with a price of 0.2 eur per kWh

v(A)=v(C)=v(D)=32.24-11.97=20.27kWh=*4.054 eur*

v(B)=8.58-4.09=4.49kWh=*0.898 eur*

In Table 15 we denote the Shapley value of all players in each round, as well as their total benefits under the pricing scheme of this case.

Table 15 Shapley value of players in an exponential increase pricing scheme

Shapley value in euros	*v(A)*	*v(B)*	*v(C)*	*v(D)*
Price 1(0.1)	*0.285*	*0.143*	*0.285*	*0.285*
Price 2(0.15)	*1.197*	*0.409*	*1.197*	*1.197*

Price 3(0.20	*4.054*	*0.898*	*4.054*	*4.054*
Total	*5.536*	*1.45*	*5.536*	*5.536*

Case 4 – Linear decrease of prices

This pricing scheme lowers the price per kWh as the amount of sold electricity decreases. So the first 60kWh will be with a price of 0.2 eur per kWh, the next 40kWh will be with 0.15 eur per kWh, and for the last 0.1 eur per kWh, when the price is flat. This makes them more motivated to sell to the community than to the grid. The difference in this case compared to the linear increase of prices case is that the first 50kWh are sold for a higher price than in case 2, thus making this pricing scheme more profitable. If the amount of electricity is inversely divided from the one in case 2, we would get the same division of benefits. Table 16 at the end denotes all players Shapley values and a final sum of benefits.

a) First 50kWh with a price of 0.2 eur per kWh

$\varphi_A = \varphi_C = \varphi_D$=(37.95*6 + 12.05*4 + 37.95*2 + 0.6*6 + 0*6)/24 = (303.6 + 48.2 + 3.6)/24 = 14.75 kwh = *2.95eur*

φ_B=(11.45*6 + 11.45*6 + 0*12)/24 = 5.725kWh = *1.145 eur*

b) Second 50 kWh with 0.15 eur per kWh price

v(A)=v(C)=v(D)=37.95-14.75=23.2kWh

v(B)=11.45-5.725=5.72kWh

$\varphi_A = \varphi_C = \varphi_D$=(23.2*6 + 23.2*6 + 3.6*2 + 21.08*4 + 0*6)/24 = (278.4 + 7.2 + 84.52)/24 = 15.41kWh = *2.31 eur*

φ_B=(5.72*6 + 5.72*6 + 3.6*6 + 0*6)/24 = (68.64 + 21.6)/24 = 3.76kWh = *0.564 eur*

c) Last 25.3 kWh with 0.1 eur per kWh price

$v(A)=v(C)=v(D)$=23.2-15.41=7.79kWh

$v(B)$=5.72-3.76=1.96kWh

$\varphi_A = \varphi_C = \varphi_D$=7.79kWh = *0.779 eur*

φ_B=1.96kWh=*0.196 eur*

Table 16 Shapley value of players in a linear decrease pricing scheme

Shapley value in euros	*v(A)*	*v(B)*	*v(C)*	*v(D)*
Price 1	*2.95*	*1.145*	*2.95*	*2.95*
Price 2	*2.31*	*0.564*	*2.31*	*2.31*
Price 3	*0.779*	*0.196*	*0.779*	*0.779*
Total	*6.039*	*1.905*	*6.039*	*6.039*

Results

Now we can compare the benefits each player incurs from different pricing schemes, as presented in Table 17. From the table we can conclude that the last case, i.e. the case of linear decrease of the price gives the highest benefits. But as stated before, this depends on the division of amount of electricity per group of different prices. In the case if the amount is divided inversely in linear decrease from the case of linear increase of prices, the highest value each user will receive would be in the third case. This is understandable since the model is made to motivate users to sell more to receive more benefits. All of these 3 cases deliver more benefits to users than the case with flat pricing scheme.

Table 17 Compared Shapley values of all players in all four cases of pricing scheme

Shapley value in euros	v(A)	v(B)	v(C)	v(D)	Total
Case 1	3.795	1.145	3.795	3.795	12.53
Case 2	5.348	1.528	5.348	5.348	17.572
Case 3	5.536	1.45	5.536	5.536	18.058
Case 4	*6.039*	*1.905*	*6.039*	*6.039*	*20.022*

5 Conclusions

As the need for using renewable resources for producing electricity increases, there will be more and more issues arising. One of them is the electricity sharing among small distributed generators, such as houses, the distribution of this electricity and motivation of users to produce it.

In this dissertation we have addressed the problem of electricity sharing among prosumers (electricity producing consumers) in a small community. The issue of fair distribution of benefits among the players is solved by the proposed Shapley theorem algorithm.

We have developed the algorithm taking into consideration the possible consumption patterns, the main source of electricity generation in the chosen area, i.e. sun exposure as well as the solar panel capacity difference and how the combination of the three accounts for the surplus/deficit of electricity each community member will have. We have considered 10 members of a community, where only 6 of them have a surplus of electricity. The model we developed, allows them to sell electricity at a specific price to the remaining 4, while sharing benefits from their collaboration in a fair way according to their contribution. Three of the producing users are considered as one player, since they have the same amount of surplus, and by acting as one, have a higher share in the coalition, thus expect higher benefits, a notion explained later in our findings. The total benefits of the coalition are 62.26 eur, considering a price of 0.1 eur per kWh. Next we consider what happens to each players' benefits when we change the consumption level of one player, while the other's consumption stays the same. As expected there is an increase of payments to the players with constant consumption, when the variable consumption of one player increases. This proves the notion of the Shapley theorem, that players with higher contribution receive the highest benefits.

Since the total surplus of electricity is more than needed in the community, we developed a model to sell the surplus of electricity to the grid according to specific pricing schemes. The total amount (125.3kWh) is sold by using one of the four pricing schemes: flat prices, linear increase, exponential increase and linear decrease of price. The electricity is sold in lots (except for the first scheme), with different prices that change according to the pricing scheme, and profits are shared fairly among all players. The pricing scheme of linear price decrease (see Table 18) turns to be the most profitable according to our electricity lot division, which is the main variable that influences the results the most. By using different division of lots, the results would be different.

Table 18 Compared results from different pricing schemes

Shapley value in euros	v(A)	v(B)	v(C)	v(D)	Total
Flat price	3.795	1.145	3.795	3.795	12.53
Linear increase	5.348	1.528	5.348	5.348	17.572
Exponential increase	5.536	1.45	5.536	5.536	18.058
Linear decrease	*6.039*	*1.905*	*6.039*	*6.039*	*20.022*

Bibliography

[1] European Commission (2006) *European SmartGrids Technology Platform:Vision and Strategy for Europe's Electricity*, http://ec.europa.eu/research/energy/pdf/smartgrids_en.pdf (accessed on 1 August 2014).

[2] U.S. Department of Energy, *Smart Grid System Report*, July 2009, http://www.oe.energy.gov/sites/prod/files/oeprod/DocumentsandMedia/SGSRMain_090707_lowres.pdf (accessed on 1 August 2014).

[3] Department of Energy and Climate Change, UK, *Smarter Grids: The Opportunity*, December 2009, http://www.techuk-e.net/Portals/0/Cache/(DECC)smart%20grid_web.pdf (accessed on 1 August 2014).

[4] Stephen J. Callahan, *Smart Grid's Evolution Starts and Ends with the Customer*, IBM, http://www.generatinginsights.com/whitepaper/smart-grids-evolution-starts-and-ends-with-the-customer.html (accessed on 10 August 2014)

[5] Michael P. Lee et al., *Assessment of Demand Response and Advanced Metering: Staff Report*, Federal Energy Regulatory Commission, October 2013

[6] Jeff St. John, *Illinois to Smart Grid: Prove you're worth it – or Pay*, December 2011 http://www.greentechmedia.com/articles/read/illinois-to-smart-grid-prove-youre-worth-it-or-pay (accessed on 20 August 2014)

[7] Robert L. Borlick, *Pricing Negawatts: DR design flaws create perverse incentives,* Public utilities fortnightly, August 2010

[8] Christopher Otieno Adika, *Automated Demand Response Approaches to Household Energy Management in a Smart Grid Environment*, University of Toledo, May 2014

[9] P. Khajavi, H.Abniki, A.B. Arani, *The Role of Incentive Based Demand Response Programs in Smart Grid*, IEEE, 2011

[10] Zubair Md. Fadlullah, Nei Kato, *Research Methods and Challenges in Demand Side Management*, IEEE Smart Grid, February 2013

[11] Li Xu, Shi-Jie Deng, *Incentive Compatible Contracting of Thermostats Control for Renewable Energy Integration*, H. Milton Stewart School of Industrial and Systems Engineering, 2013

[12] G. Koutitas, *Control of Flexible Smart Devices in the Smart Grid*, IEEE Transactions on Smart Grid, 2012

[13] *Advanced Metering Infrastructure (AMI)*, Electric Power Research Institute, February 2007

[14] Papadimitriou C.N., Vovos N.A, *Fuzzy control of WT with DFIG for Integration into Micro-grids, Fundamental and Advanced Topics in Wind Power*, June 2011

[15] Soultanis N.L., *Contribution to the control and simulation of low voltage power systems with distributed generation*, NTUA, 2008

[16] Katiraei F., Iravani M.R., Lehn P.W., *Micro-grid Autonomous Operation During and Subsequent to Islanding Process*, IEEE transactions on power delivery, January 2005

[17] Bakken D., *Smart Grids: Clouds Communications, Open Source and Automation*, CRC Press, 2014

[18] Ekanayake J. et al., *Smart Grid: Technology and applications*, John Wiley&Sons Ltd, 2012

[19] Lacey S., *Rise of the Prosumer: Will Homeowners Ever Be More Important Than Power Plants*, June 2014, http://www.greentechmedia.com/articles/read/rise-of-the-electricity-prosumer, (accessed on 8 September 2014)

[20] Lacey S., *Storage Is The New Solar: Will Batteries and PV Create and Unstoppable Hybrid Force?*, June 2014, http://www.greentechmedia.com/articles/featured/Storage-Is-the-New-Solar-Will-Batteries-and-PV-Create-an-Unstoppable-Hybri, (accessed on 8 September 2014)

[21] Grijalva S., Tariq M.U., *Prosumer-based smart grid architecture enables a flat, sustainable electricity industry*, IEEE, January 2011

[22] Kennedy J., *Transactive energy: the next big deal for the smart grid*, July 2013, http://www.yourenergyblog.com/transactive-energy-the-next-big-deal-for-the-smart-grid/ (accessed 25 September 2014)

[23] Cazalet E.G., *TeMIX: A Foundation for Transactive Energy in a Smart Grid World*, Grid-Interop Forum, 2010

[24] H.L. Willis, L. Phillipson, *Understanding electric utilities and De-regulation*, 1998

[25] US-Canada Power System Outage Task Force, *Final Report on the August 14, 2003 Blackout in the United States and Canada: Causes and Recommendations*, April 2004

[26] J.D. Glover, M.S. Sarma, *Power systems analysis and design*, May 2007

[27] Brandenburger A., *Cooperative Game Theory: Characteristic Functions, Allocations, Marginal Contributions*, 2007

[28] Brown K.L, *Coalitional game theory: Definitions*, November 2013, https://www.youtube.com/watch?v=lJVmR7CmLeQ, (accessed 5 October 2014)

[29] R. A. McCain *A Nontechnical Introduction to the Analysis of Strategy Revised Edition*, August 2010

[30] G. Owen, *Game Theory*, October 1995

[31] R. Serrano, *Cooperative games: Core and Shapley value*, July 2007

[32] Brown K.L, *The Core*, November 2013, https://www.youtube.com/watch?v=DW_-I8UuU6I&index=4&list=PLeY-lFPWgBTif4PmLSN8eJsfOhFv_QUPl, (accessed 5 October 2014)

[33] R. Weber, *Cooperative Games*, Lecture notes, University of Cambridge, 2014 http://www.statslab.cam.ac.uk/~ff271/teaching/mor/notes/notes19.pdf (accessed 11 October)

[34] H. Aziz, F. Brandt, P. Harrenstein, *Monotone cooperative games and their threshold versions*, University of Munich, 2010

[35] S.C. Littlechild, G. Owen, *A simple expression for the Shapley value in a special case*, Management Science, November 1973

[36] M. Jackson, The Shapley value, November 2013, https://www.youtube.com/watch?v=qcLZMYPdpH4&list=UU73W1eqdxKPwr2OlfkMhQkQ, (accessed 5 October 2014)

[37] T.S. Ferguson, Game Theory: Games in Coalitional Form, http://www.math.ucla.edu/~tom/Game_Theory/coal.pdf, (accessed 15 October 2014)

[38] A. E. Roth, *The Shapley value*, Cambridge University press, 1988